Data based modelling of electron fluxes at GEO and statistical wave models

R. J. Boynton

University of Sheffield, UK.

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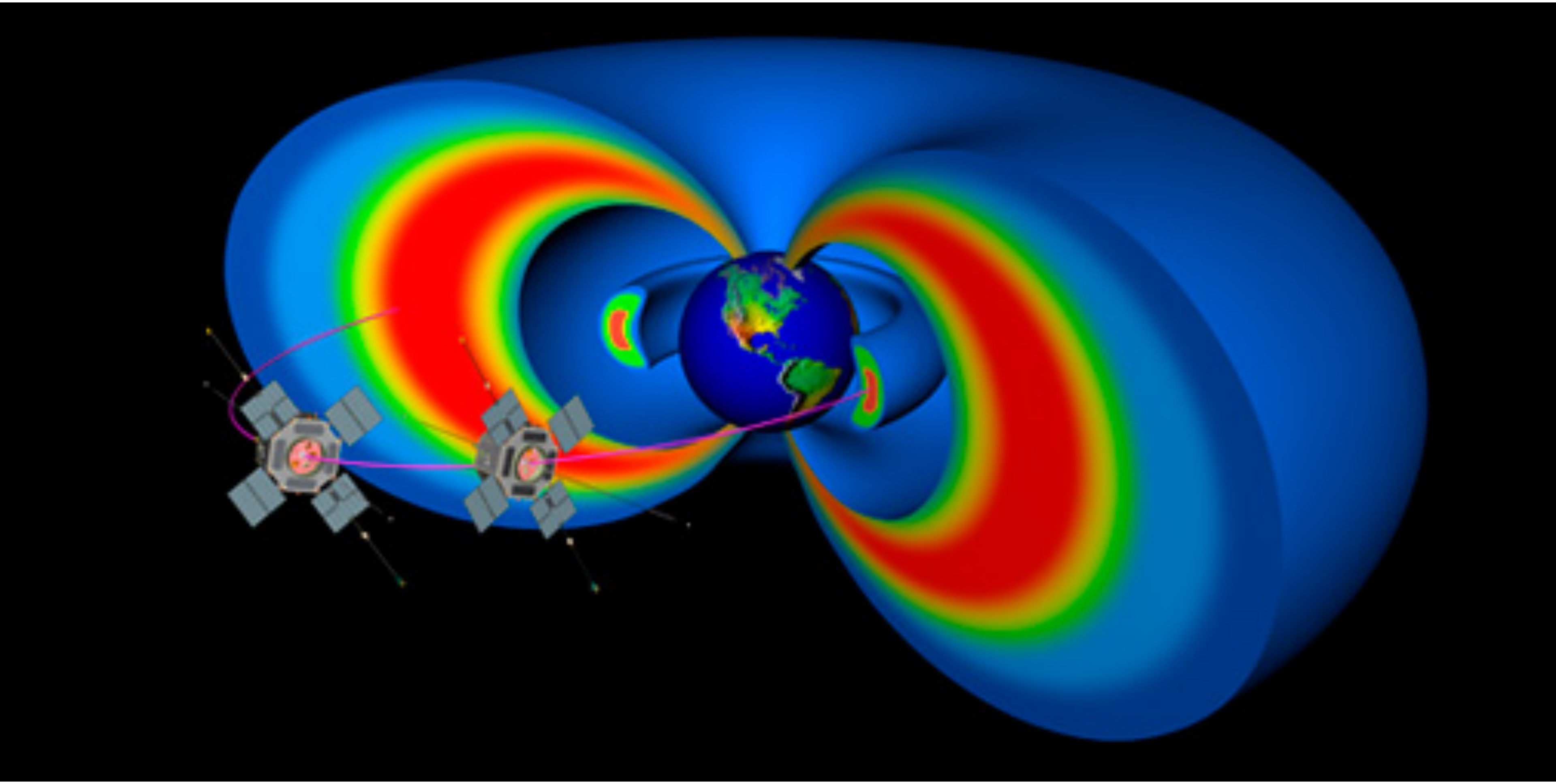
Data based modelling of radiation belt electron fluxes at GEO



Radiation Belts

Between 1985-2012 there have been 19 serious incidences

Five of which resulted in a total loss of the satellite

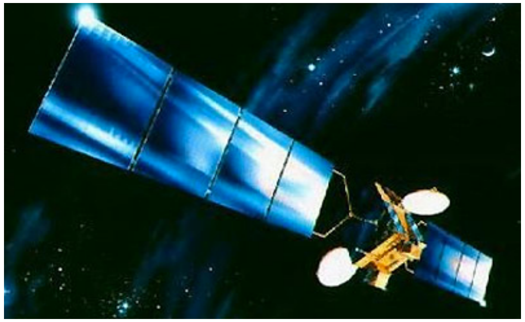
Manufacture costs/satellite
$250 - $350 M

Lost revenue /satellite
~$ 150 M/year

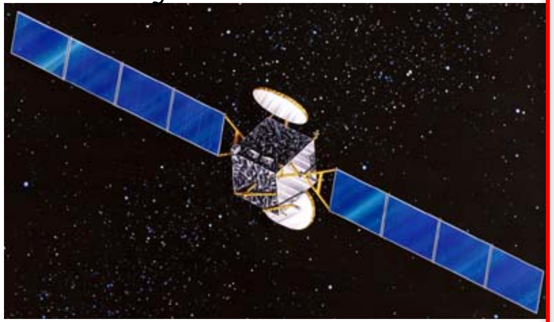
Satellite lifetime
15-20 years

Irrecoverable Loss of Satellite

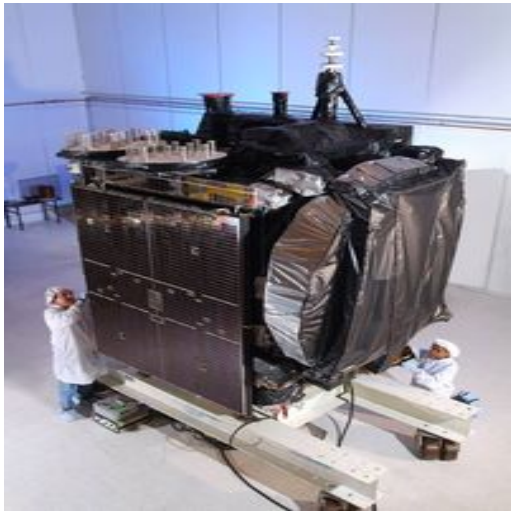
Telstar 401



Galaxy 4



Galaxy 15

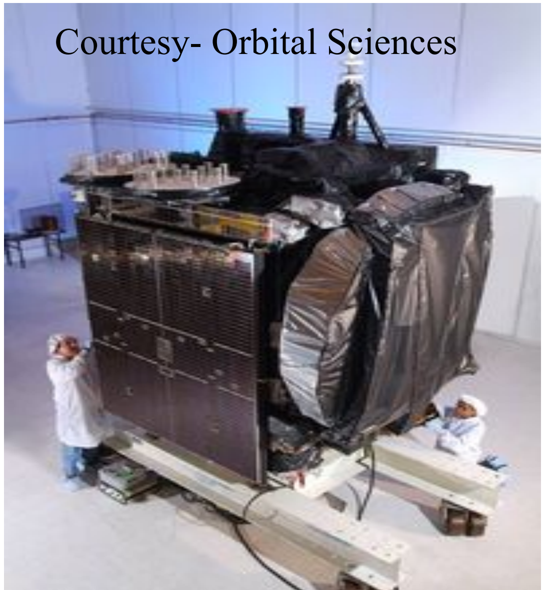


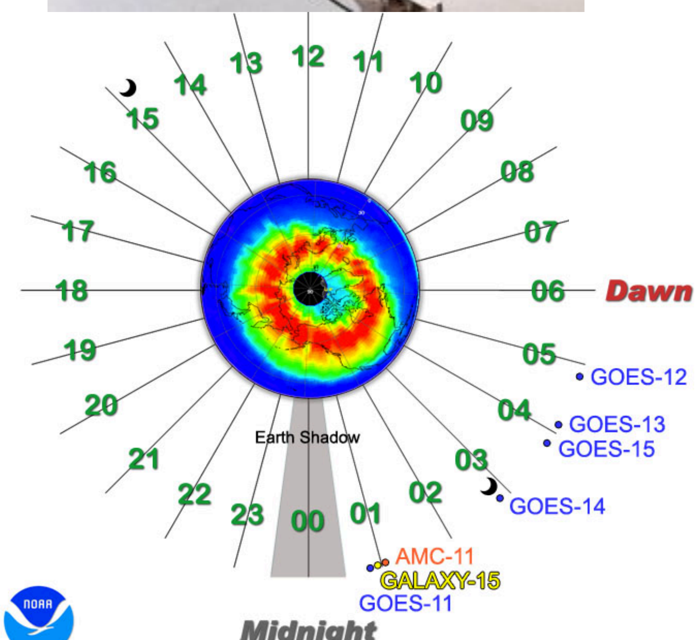
Unresponsive to ground control commands

Interfered with other communication satellites

Recovered after 1 year

Spacecraft in the Radiation Belts


Courtesy- Orbital Sciences



Galaxy 15

Geostationary communication satellite

Became unresponsive to commands after a small space weather event and began to drift

Galaxy 15′s telecommunications remained fully functional

This could have interfered with the AMC-11 satellite that distributes television throughout the USA

The effects of space weather: Radiation Belts

We need to be able to forecast the times when the radiation belt environment will be hazardous to the spacecraft to help satellite operators mitigate any issues arise with the spacecraft.

To forecast these events we need a reliable model of the radiation belts

Aims

Work Package 6 of PROGRESS is devoted to the development of models that are able to forecast the electron radiation in the radiation belts.

Modelling

First principles vs. System identification approach

Standard Approach

Physical Knowledge

+

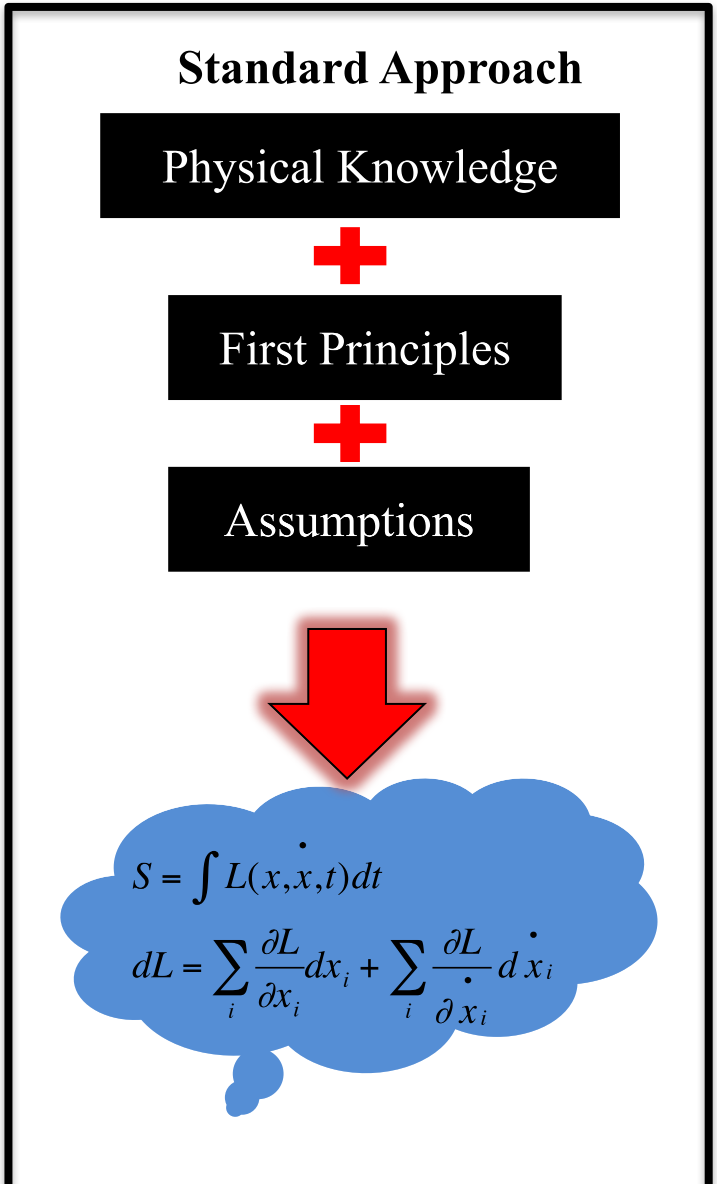
First Principles

+

Assumptions

Modelling

First principles vs. System identification approach



Modelling

First principles vs. System identification approach

Standard Approach

Physical Knowledge

+

First Principles

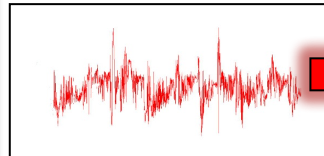
+

Assumptions

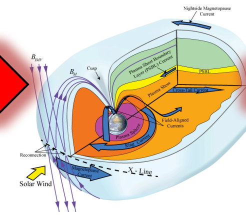
$$S = \int L(x,\dot{x},t)dt$$

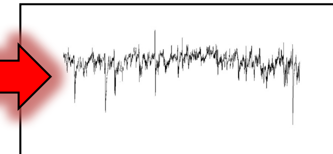
$$dL = \sum_i \frac{\partial L}{\partial x_i} dx_i + \sum_i \frac{\partial L}{\partial \dot{x}_i} d\dot{x}_i$$

Systems Approach



Input Data





Output Data

Modelling

First principles vs. System identification approach

Standard Approach

Physical Knowledge

+

First Principles

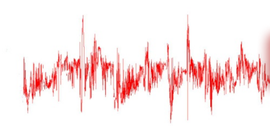
+

Assumptions

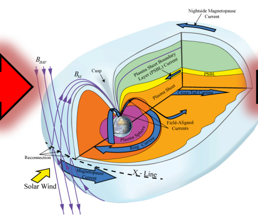
$$S = \int L(x, \dot{x}, t)\, dt$$

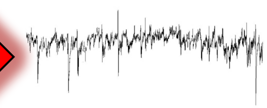
$$dL = \sum_i \frac{\partial L}{\partial x_i} dx_i + \sum_i \frac{\partial L}{\partial \dot{x}_i} d\dot{x}_i$$

Systems Approach



Input Data



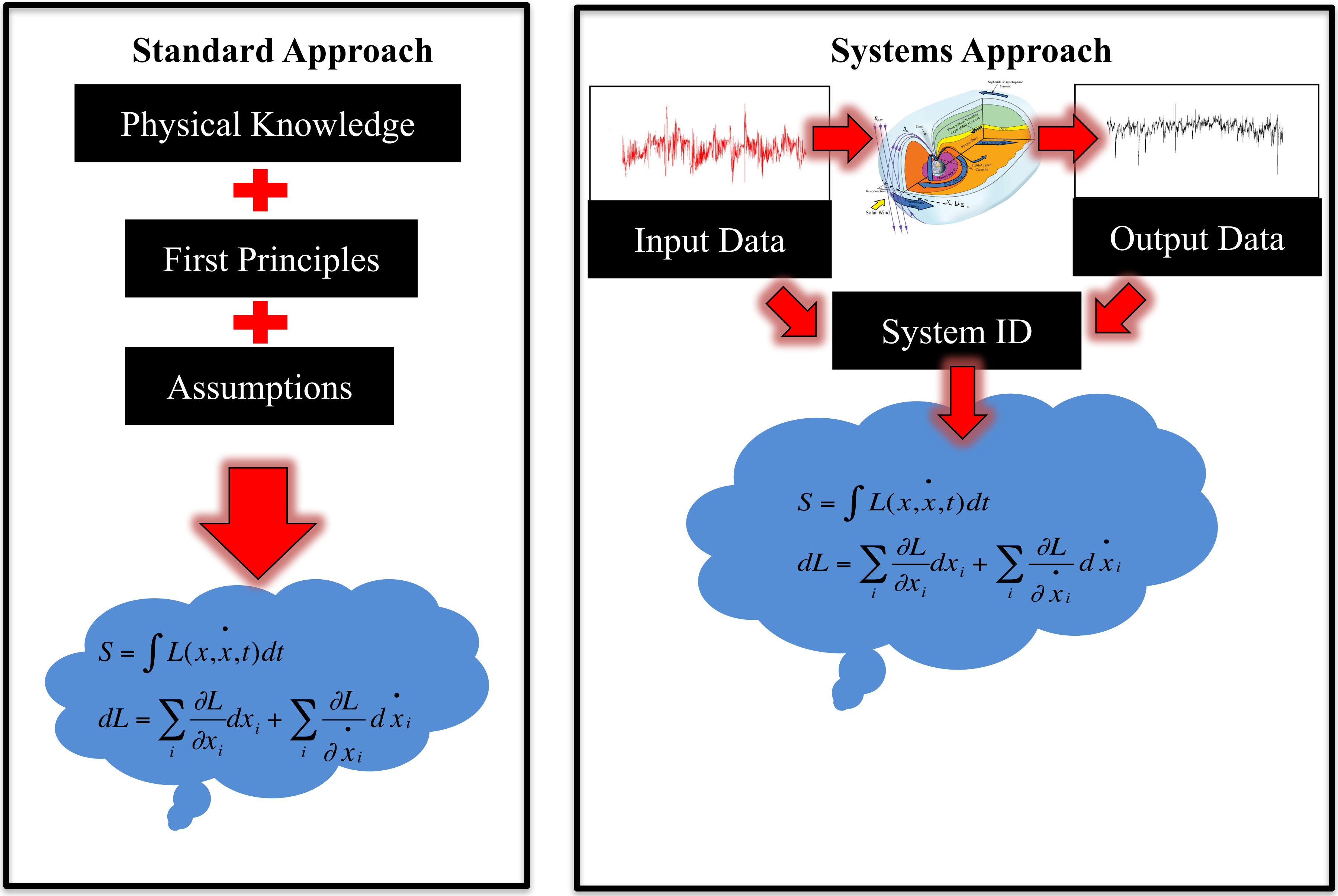


Output Data

System ID

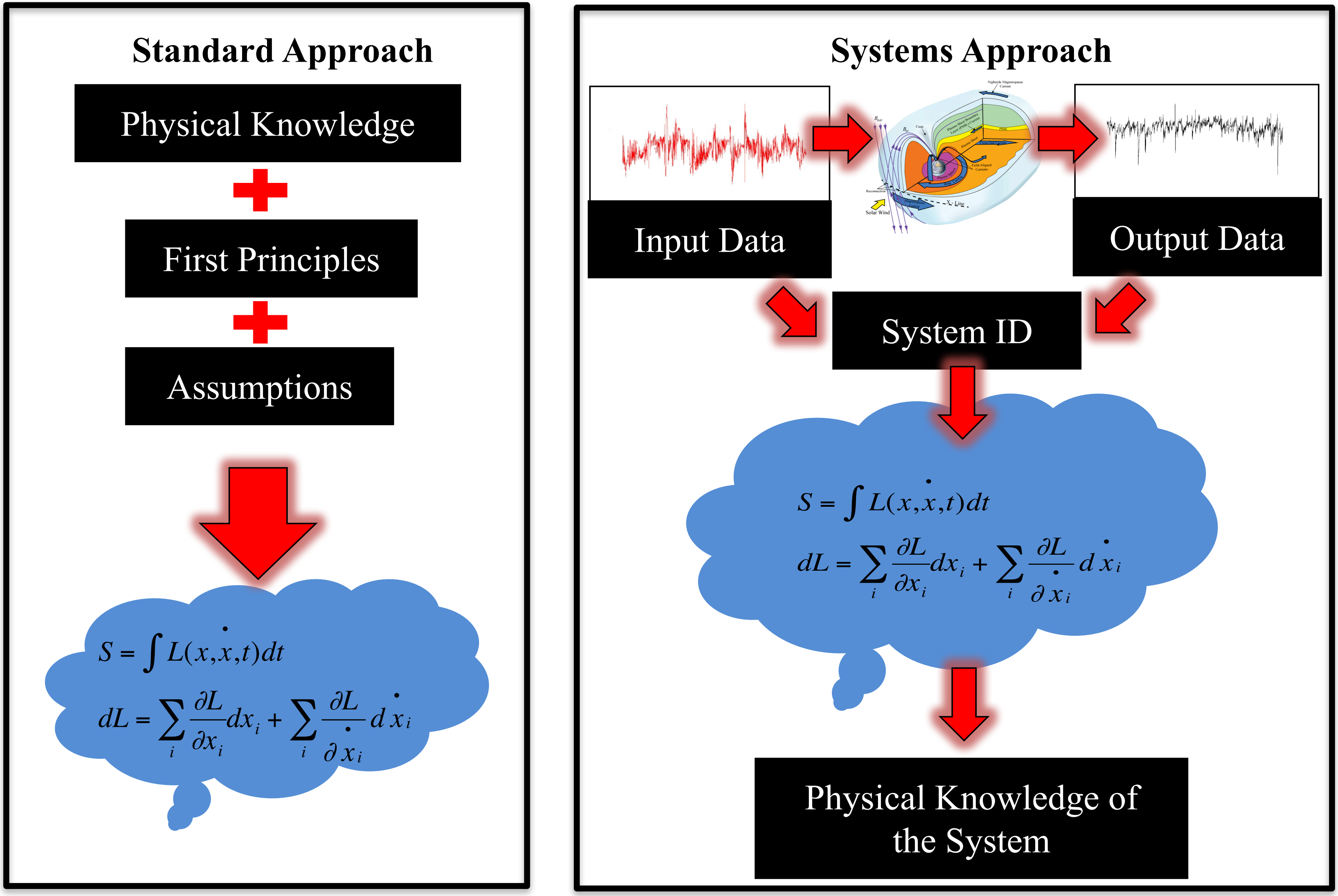
Modelling

First principles vs. System identification approach



Modelling

First principles vs. System identification approach

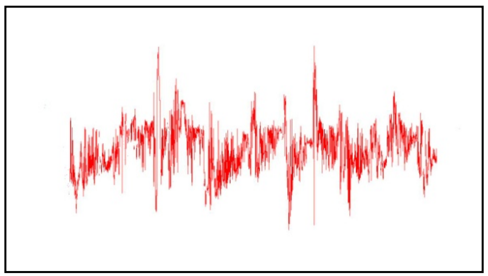


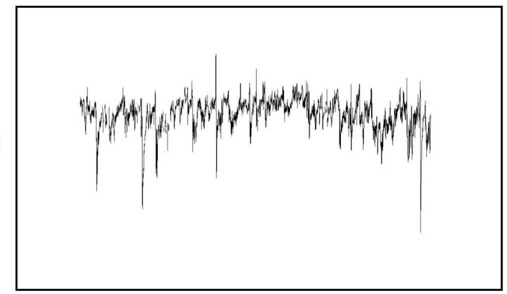
System Identification

Input to the system, u(t) | **System** | **Output measurement, y(t)**

System Identification

Input to the system, u(t) | **System** | **Output measurement, y(t)**





Mapping the input to the output

- Neural Networks
- Genetic Algorithms
- Linear Prediction Filters
- NARMAX – Physically Interpretable

NARMAX

Nonlinear

$$
\begin{aligned}
y(t) = F[&y(t-1),\ldots y(t-n_y),\\
&u_1(t-1),\ldots,u_1(t-n_{u_1}),\ldots,\\
&u_m(t-1),\ldots,u_m(t-n_{u_m}),\\
&e(t-1),\ldots,e(t-n_e)] + e(t)
\end{aligned}
$$

NARMAX

Nonlinear **AutoRegressive**

$$
\begin{aligned}
y(t) = F[\,& y(t-1),\ldots y(t-n_y), \\
& u_1(t-1),\ldots,u_1(t-n_{u_1}),\ldots, \\
& u_m(t-1),\ldots,u_m(t-n_{u_m}), \\
& e(t-1),\ldots,e(t-n_e)] + e(t)
\end{aligned}
$$

NARMAX

Nonlinear **A**uto**R**egressive **M**oving **A**verage

$$y(t) = F[y(t-1),\ldots y(t-n_y),$$
$$u_1(t-1),\ldots,u_1(t-n_{u_1}),\ldots,$$
$$u_m(t-1),\ldots,u_m(t-n_{u_m}),$$
$$e(t-1),\ldots,e(t-n_e)] + e(t)$$

NARMAX

Nonlinear **A**uto**R**egressive **M**oving **A**verage with e**X**ogenous inputs

$$
\begin{aligned}
y(t) = F[\,& y(t-1),\ldots y(t-n_y), \\
& u_1(t-1),\ldots,u_1(t-n_{u_1}),\ldots, \\
& u_m(t-1),\ldots,u_m(t-n_{u_m}), \\
& e(t-1),\ldots,e(t-n_e)] + e(t)
\end{aligned}
$$

NARMAX

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&u_1(t-1),\ldots,u_1(t-n_{u_1}),\ldots,\\
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&e(t-1),\ldots,e(t-n_e)] + e(t)
\end{aligned}
$$

NARMAX Model:

- Nonlinear Function *F*. e.g. Polynomial, Wavelets, etc.
  - Degree of polynomial
  - Type of wavelet
- Inputs
- System lags

NARMAX

Nonlinear **A**uto**R**egressive **M**oving **A**verage with e**X**ogenous inputs

$$y(t) = F[y(t-1),\dots y(t-n_y), \\ u_1(t-1),\dots,u_1(t-n_{u_1}),\dots, \\ u_m(t-1),\dots,u_m(t-n_{u_m}), \\ e(t-1),\dots,e(t-n_e)] + e(t)$$

NARMAX Model:

- Nonlinear Function *F*. e.g. Polynomial, Wavelets, etc.
  - Degree of polynomial
  - Type of wavelet
- Inputs
- System lags

Polynomial

- FROLS algorithm

  Involves three stages

  1. Structure selection: Error Reduction Ratio (ERR)
  2. Coefficient estimation
  3. Model validation

NARMAX FROLS

Structure Selection

Expand Nonlinear Function F → Term Dictionary

$$[y(t-1),\ldots,u_4(t-4),\ldots,y^2(t-1),\ldots,y^2(t-n_y),\ldots$$
$$y(t-1)u_1(t-1),\ldots,y(t-1)u_1(t-n_{u_1}),\ldots,$$
$$u_3(t-2)u_4(t-1),\ldots,u_3(t-2)u_4(t-n_{u_m}),\ldots$$
$$e(t-1),\ldots,u_2(t-3)e(t-n_e),\ldots,e^2(t-n_e)]$$

NARMAX FROLS

Structure Selection

Expand Nonlinear Function F

Term Dictionary

$$[y(t-1),\ldots,u_4(t-4),\ldots,y^2(t-1),\ldots,y^2(t-n_y),\ldots$$
$$y(t-1)u_1(t-1),\ldots,y(t-1)u_1(t-n_{u_1}),\ldots,$$
$$u_3(t-2)u_4(t-1),\ldots,u_3(t-2)u_4(t-n_{u_m}),\ldots$$
$$e(t-1),\ldots,u_2(t-3)e(t-n_e),\ldots,e^2(t-n_e)]$$

Calculate ERR wrt output $y(t)$

Term	ERR
$y(t-1)$	0.565
$\vdots$	
$u_1(t-1)$	0.239
$\vdots$	
$u_3(t-2)u_4(t-1)$	0.784
$\vdots$	
$u_3(t-2)$	0.745
$\vdots$	
$u_4^2(t-4)$	0.003

1st Stage

NARMAX FROLS

Structure Selection

Expand Nonlinear Function F

Term Dictionary

$$[y(t-1),\ldots,u_4(t-4),\ldots,y^2(t-1),\ldots,y^2(t-n_y),\ldots$$
$$y(t-1)u_1(t-1),\ldots,y(t-1)u_1(t-n_{u_1}),\ldots,$$
$$u_3(t-2)u_4(t-1),\ldots,u_3(t-2)u_4(t-n_{u_m}),\ldots$$
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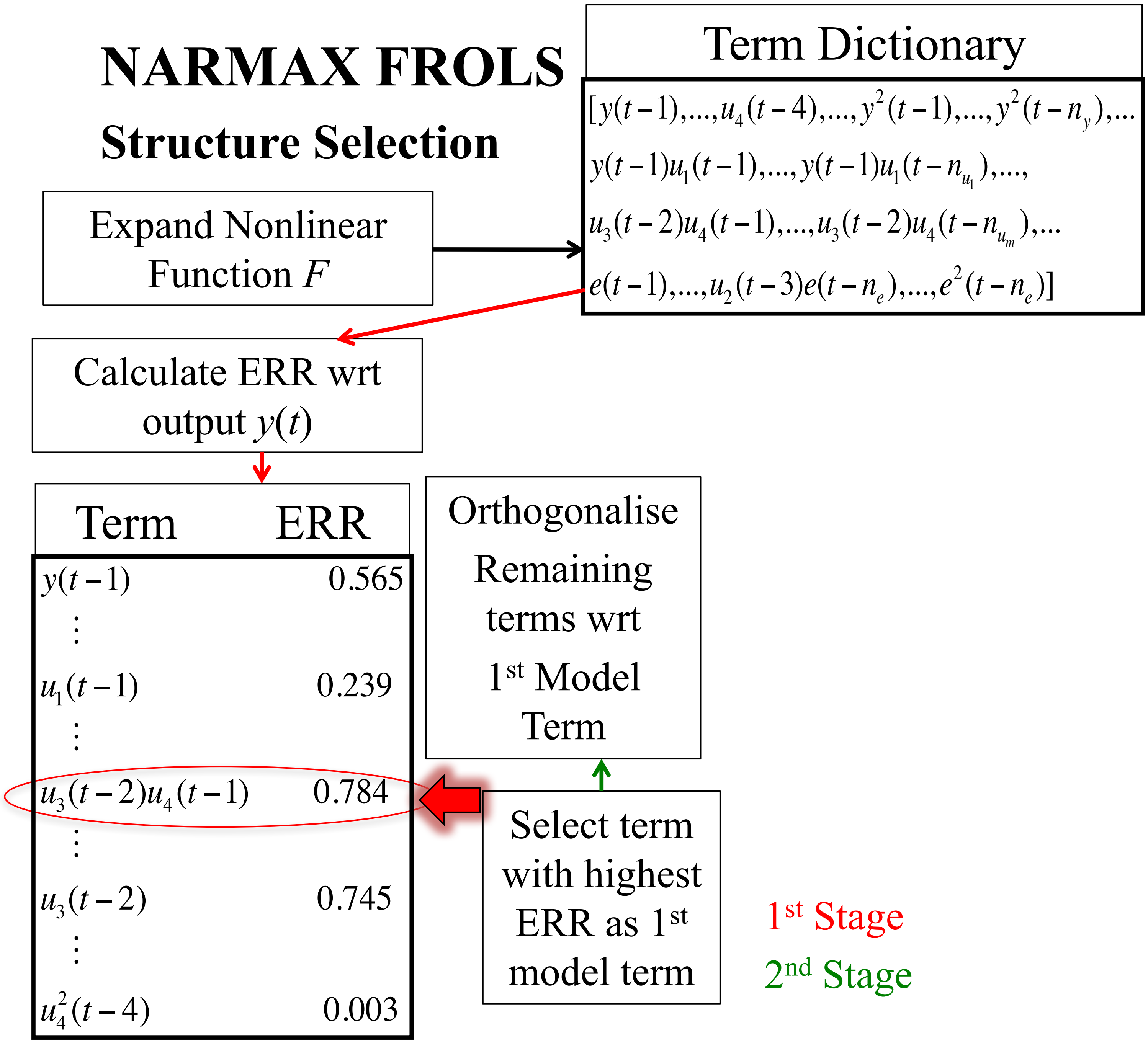
$\vdots$	
$u_3(t-2)$	0.745
$\vdots$	
$u_4^2(t-4)$	0.003

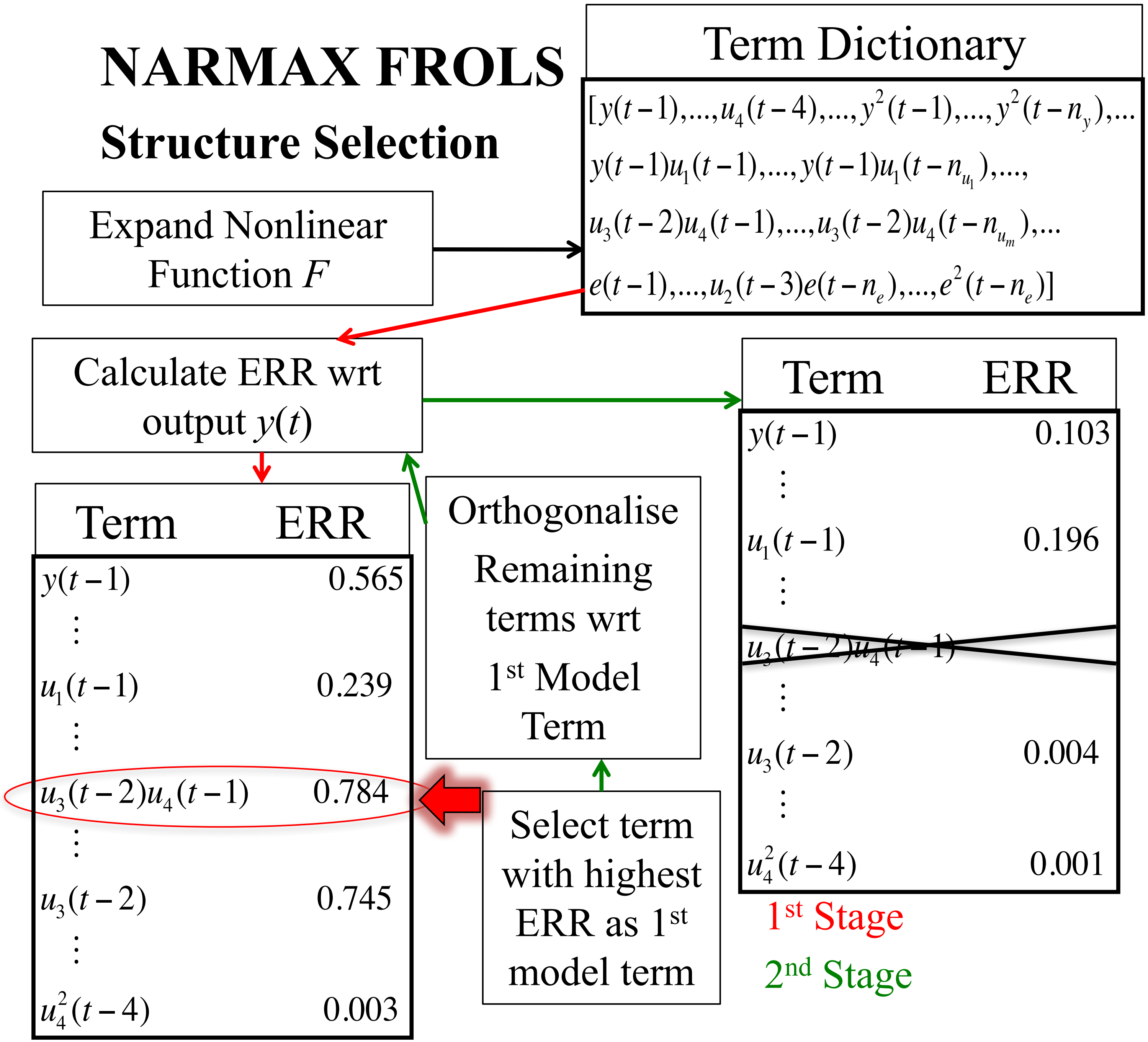
Select term with highest ERR as 1st model term

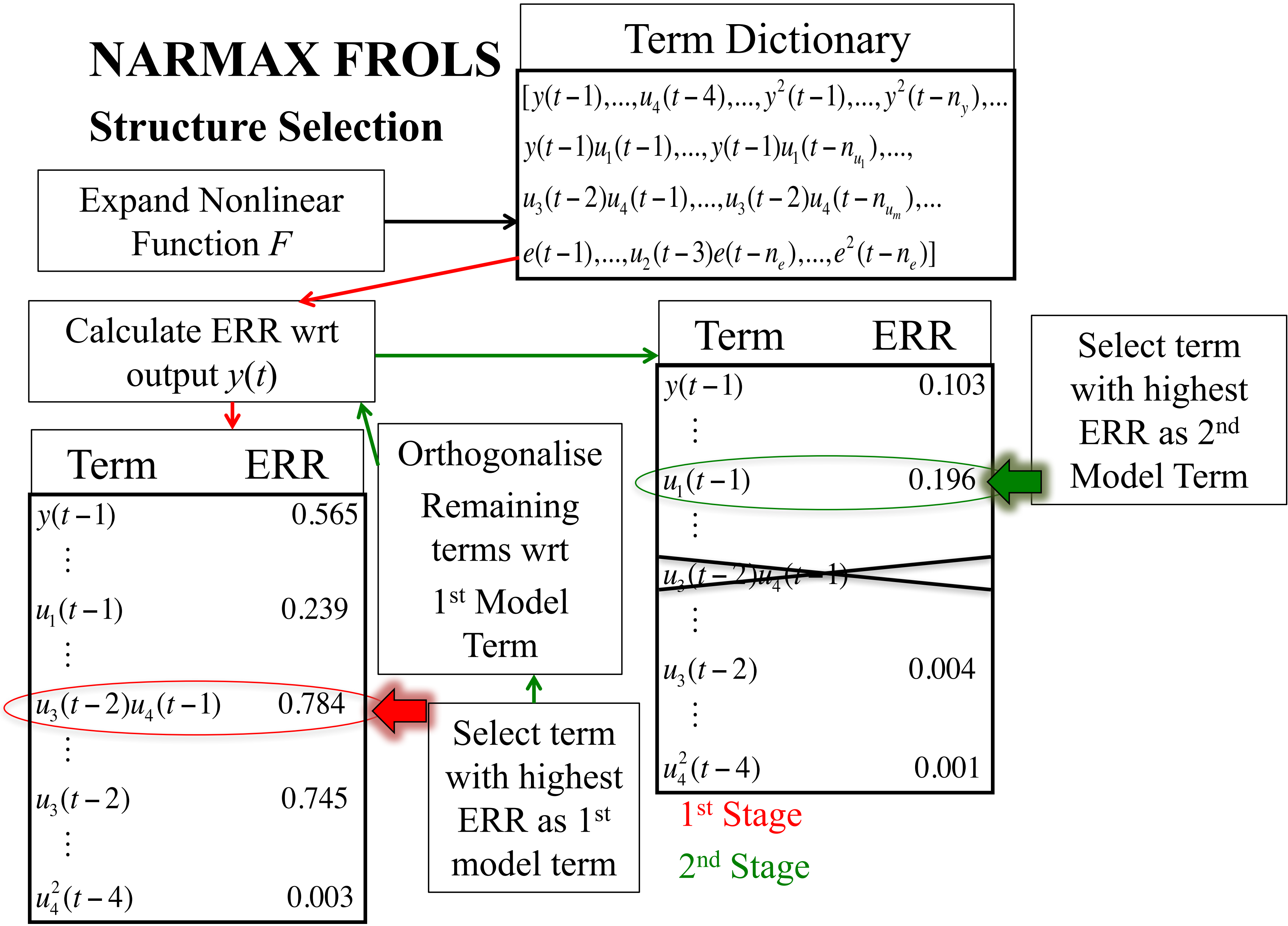
1st Stage

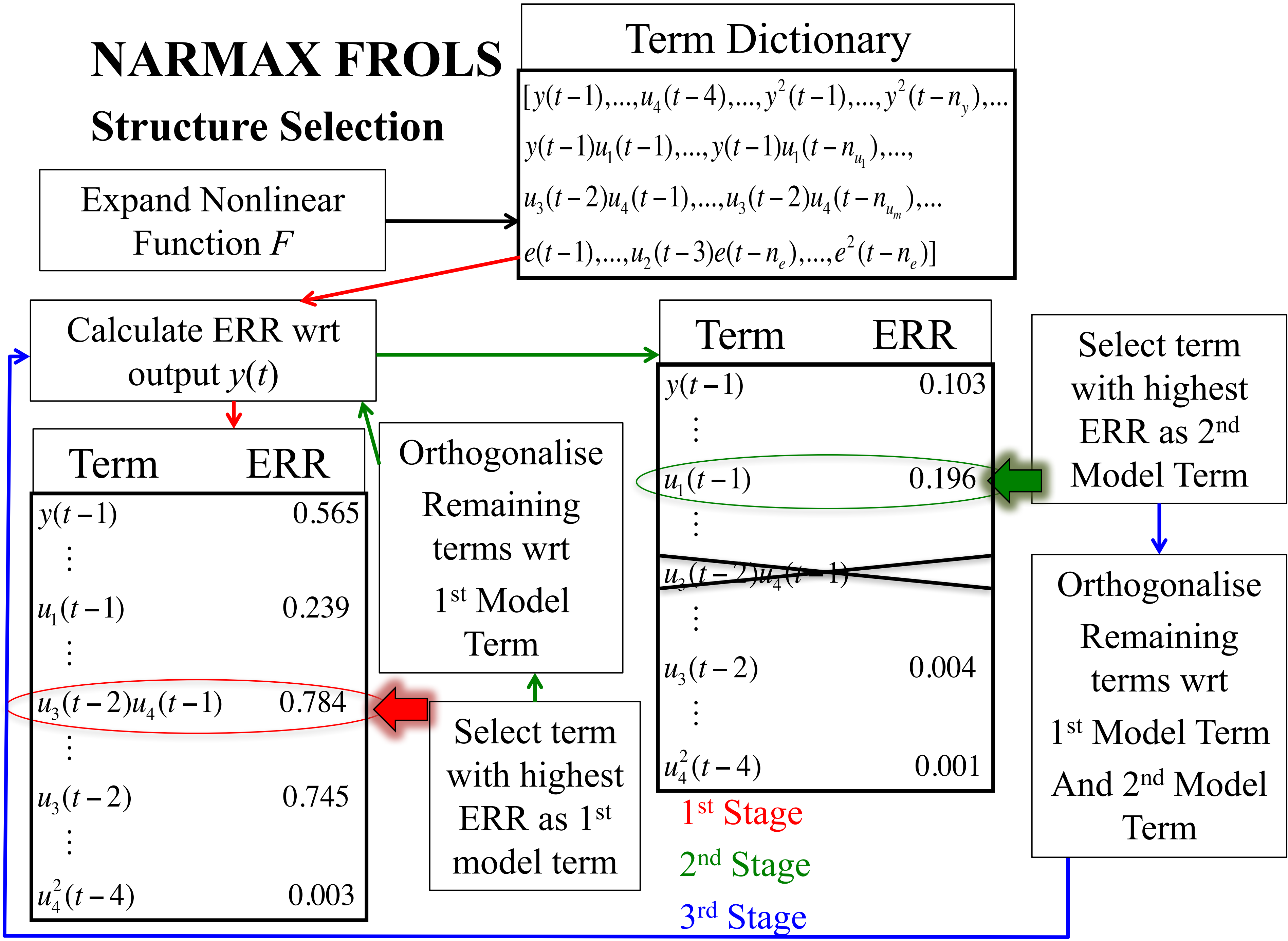
NARMAX FROLS

Structure Selection




NARMAX FROLS
Structure Selection
Term Dictionary
[y(t−1),...,u_4(t−4),...,y^2(t−1),...,y^2(t−n_y),...
y(t−1)u_1(t−1),...,y(t−1)u_1(t−n_{u_1}),...,
u_3(t−2)u_4(t−1),...,u_3(t−2)u_4(t−n_{u_m}),...
e(t−1),...,u_2(t−3)e(t−n_e),...,e^2(t−n_e)]
Expand Nonlinear Function F
Calculate ERR wrt output y(t)
Term ERR
y(t−1) 0.565
u_1(t−1) 0.239
u_3(t−2)u_4(t−1) 0.784
u_3(t−2) 0.745
u_4^2(t−4) 0.003
Orthogonalise Remaining terms wrt 1st Model Term
Select term with highest ERR as 1st model term
Term ERR
y(t−1) 0.103
u_1(t−1) 0.196
u_3(t−2)u_4(t−1)
u_3(t−2) 0.004
u_4^2(t−4) 0.001
1st Stage
2nd Stage


NARMAX FROLS
Structure Selection
Term Dictionary
$[y(t-1),\ldots,u_4(t-4),\ldots,y^2(t-1),\ldots,y^2(t-n_y),\ldots$
$y(t-1)u_1(t-1),\ldots,y(t-1)u_1(t-n_{u_1}),\ldots,$
$u_3(t-2)u_4(t-1),\ldots,u_3(t-2)u_4(t-n_{u_m}),\ldots$
$e(t-1),\ldots,u_2(t-3)e(t-n_e),\ldots,e^2(t-n_e)]$
Expand Nonlinear Function F
Calculate ERR wrt output $y(t)$
Term ERR
$y(t-1)$ 0.565
$u_1(t-1)$ 0.239
$u_3(t-2)u_4(t-1)$ 0.784
$u_3(t-2)$ 0.745
$u_4^2(t-4)$ 0.003
Orthogonalise Remaining terms wrt 1st Model Term
Select term with highest ERR as 1st model term
Term ERR
$y(t-1)$ 0.103
$u_1(t-1)$ 0.196
$u_3(t-2)u_4(t-1)$
$u_3(t-2)$ 0.004
$u_4^2(t-4)$ 0.001
Select term with highest ERR as 2nd Model Term
1st Stage
2nd Stage



NARMAX FROLS
Structure Selection
Term Dictionary
$[y(t-1),...,u_4(t-4),...,y^2(t-1),...,y^2(t-n_y),...$
$y(t-1)u_1(t-1),...,y(t-1)u_1(t-n_{u_1}),...,$
$u_3(t-2)u_4(t-1),...,u_3(t-2)u_4(t-n_{u_m}),...$
$e(t-1),...,u_2(t-3)e(t-n_e),...,e^2(t-n_e)]$
Expand Nonlinear Function F
Calculate ERR wrt output $y(t)$
Term ERR
$y(t-1)$ 0.565
$u_1(t-1)$ 0.239
$u_3(t-2)u_4(t-1)$ 0.784
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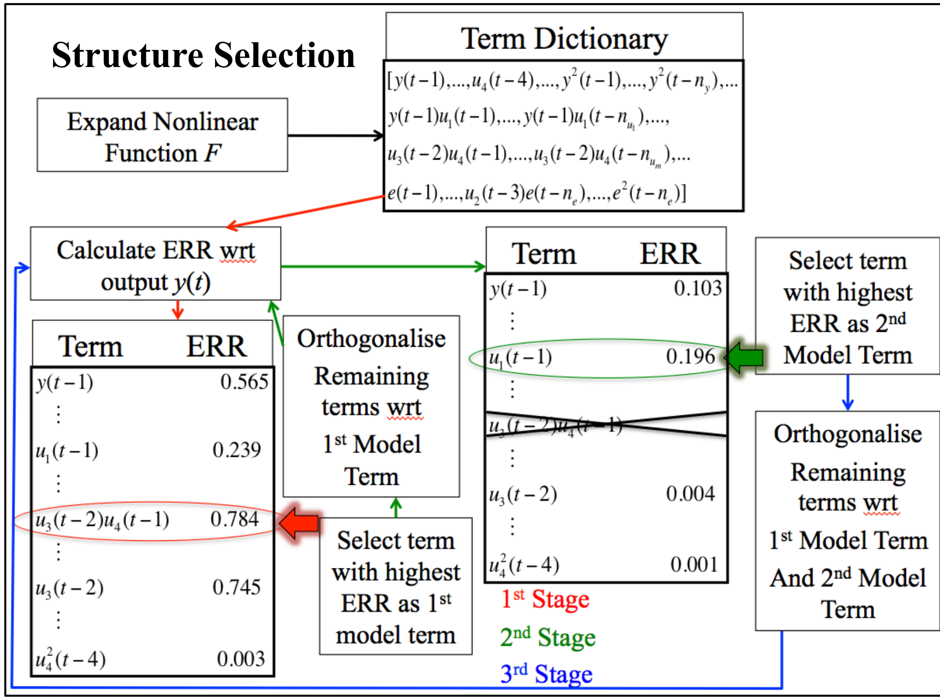
$u_4^2(t-4)$ 0.003
Orthogonalise Remaining terms wrt 1st Model Term
Select term with highest ERR as 1st model term
Term ERR
$y(t-1)$ 0.103
$u_1(t-1)$ 0.196
$u_3(t-2)u_4(t-1)$
$u_3(t-2)$ 0.004
$u_4^2(t-4)$ 0.001
1st Stage
2nd Stage
3rd Stage
Select term with highest ERR as 2nd Model Term
Orthogonalise Remaining terms wrt 1st Model Term And 2nd Model Term

NARMAX FROLS
Coefficient Estimation



$u_3(t-2)u_4(t-1)$

$u_1(t-1)$

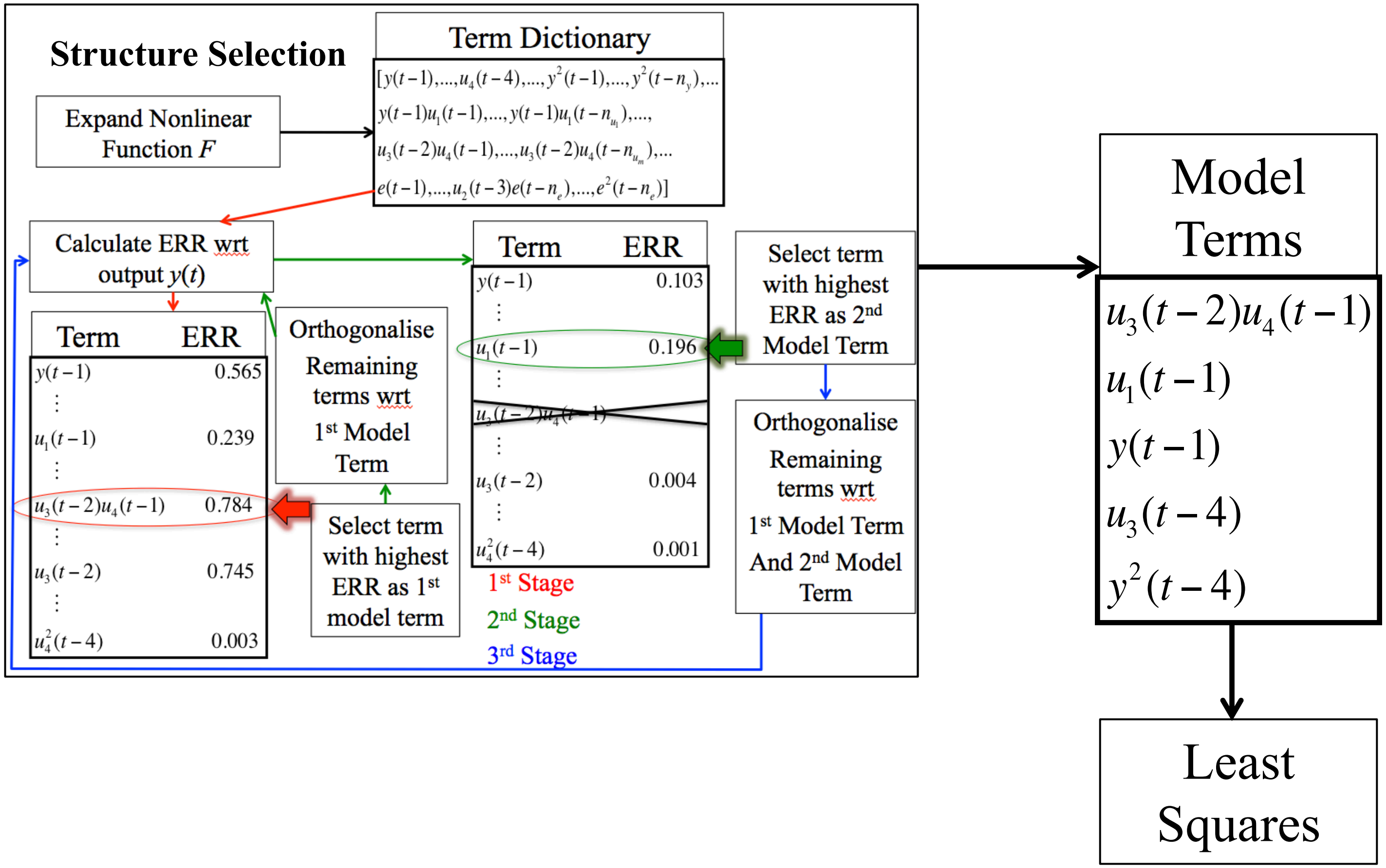
$y(t-1)$

$u_3(t-4)$

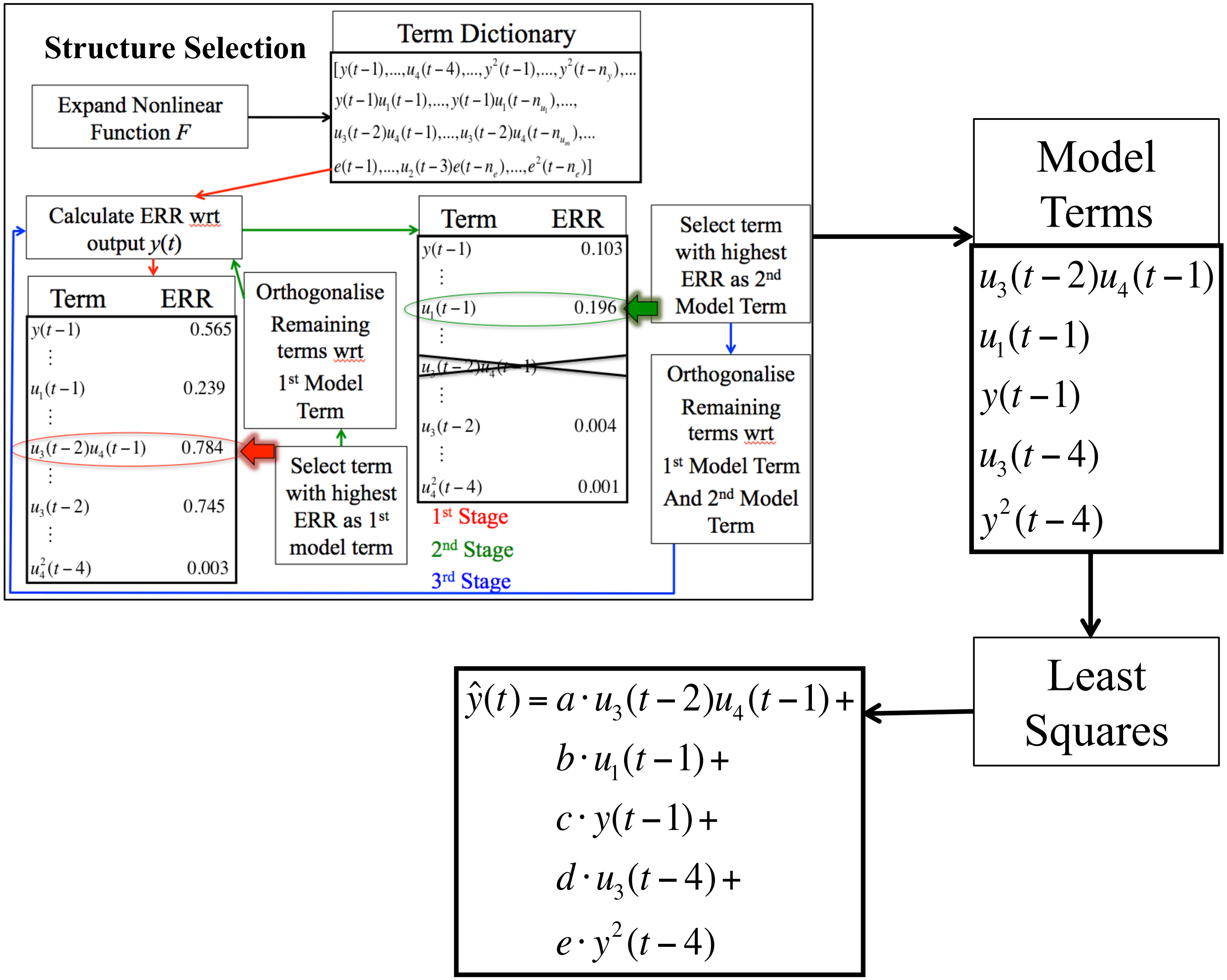
$y^2(t-4)$

NARMAX FROLS

Coefficient Estimation



NARMAX FROLS
Coefficient Estimation



Electron Flux Models

A separate NARMAX model was developed for the >800 keV and >2 MeV energies using:

Output Data

GOES Electron Fluxes J

Lags: 24 hours, 48 hours

Input Data

Solar wind Velocity V, Density n,

the Dst Index Dst, z IMF Bz, and

the time IMF was southward per day τ_{Bz}.

Lags: 24 hours, 48 hours

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Lags: 24 hours, 48 hours

$$
\begin{aligned}
J(t) \quad = \quad & F[J(t-24h), J(t-48h), \\
& V(t-24h), V(t-48h), \\
& n(t-24h), n(t-48h), \\
& B_z(t-24h), B_z(t-48h), \\
& \tau_{B_z}(t-24h), \tau_{B_z}(t-48h), \\
& Dst(t-24h), Dst(t-48h), \\
& e(t-24h), e(t-48h)]
\end{aligned}
$$

F is a third degree polynomial

Electron Flux Models - Performance

The performance of the model was assessed using the Correlation Coefficient (CC)

$$CC = \frac{\sum_{t=1}^{N}\left[\left(y(t)-\bar{y}(t)\right)\left(\hat{y}(t)-\bar{\hat{y}}(t)\right)\right]}{\sqrt{\sum_{t=1}^{N}\left[\left(y(t)-\bar{y}(t)\right)^{2}\right]\sum_{t=1}^{N}\left[\left(\hat{y}(t)-\bar{\hat{y}}(t)\right)^{2}\right]}}$$

and Prediction Efficiency (PE)

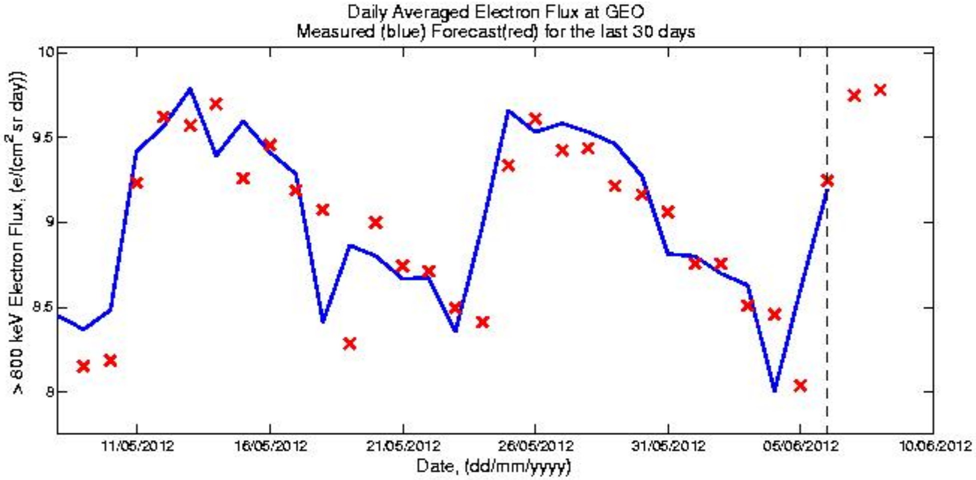
$$PE = 1-\frac{\sum_{t=1}^{N}\left[\left(y(t)-\hat{y}(t)\right)^{2}\right]}{\sum_{t=1}^{N}\left[\left(y(t)-\bar{y}(t)\right)^{2}\right]}$$

Where $y(t)$ is the measured output at time t, $\hat{y}$ is the forecast output, N is the length of the data and the bar indicates the mean.

Electron Flux Model – SNB^{3}GEO

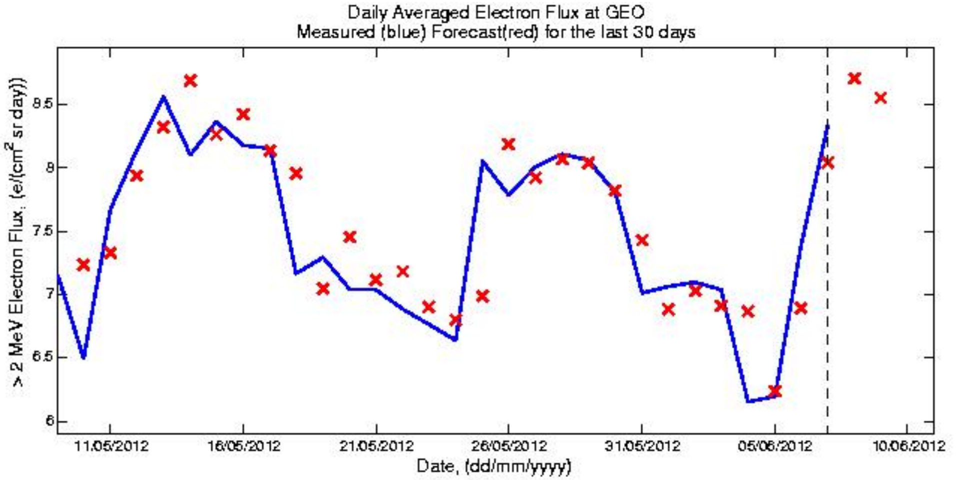
>800 keV Electron flux model at geosynchronous orbit

PE = 0.700 and CC = 0.847 for 18 months of data between 01/01/2011 30/06/2012

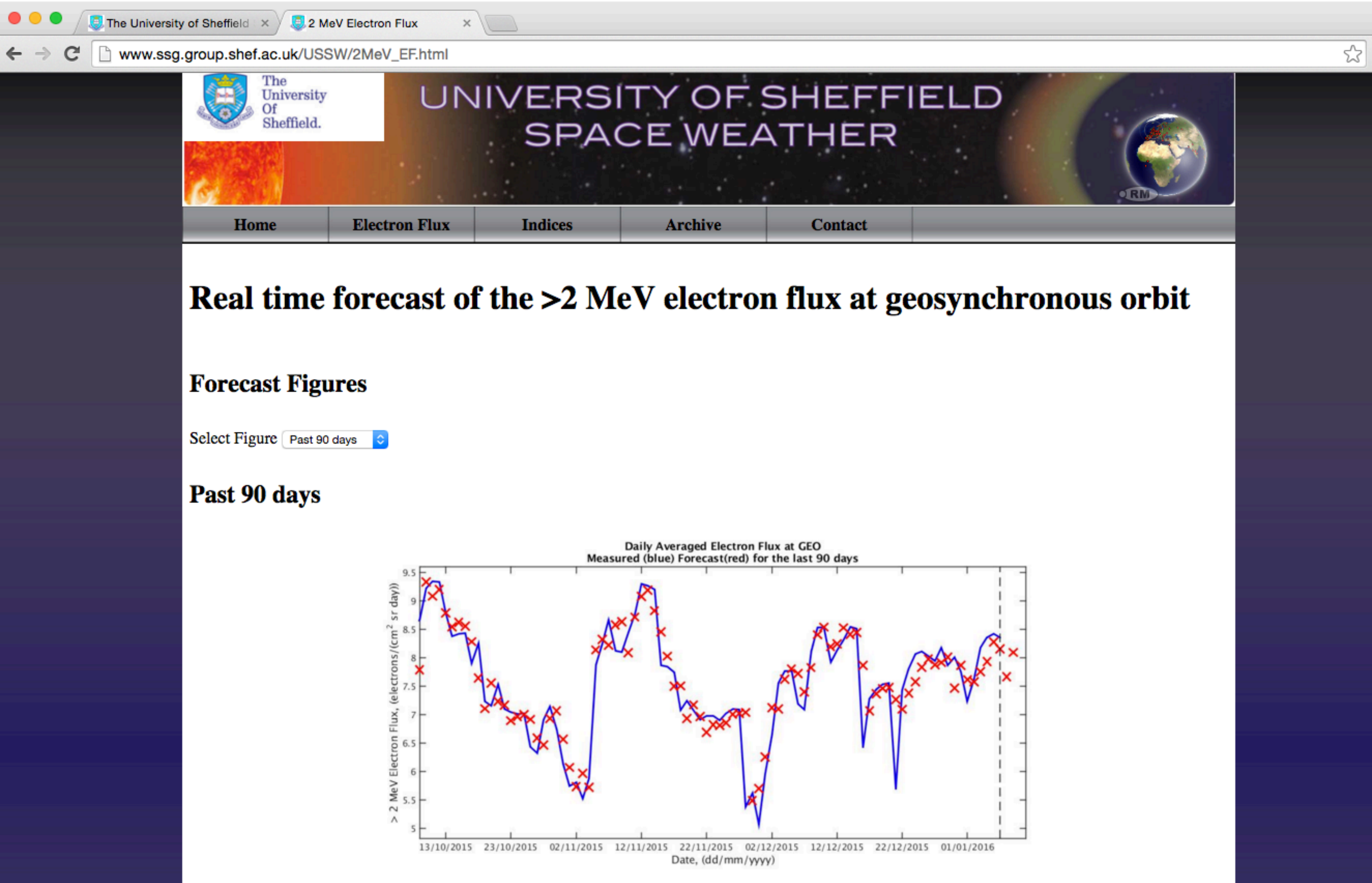


>2 MeV Electron flux model at geosynchronous orbit

PE = 0.786 and CC = 0.894 for over 26 months of data between 14/04/2010 to 30/06/2012

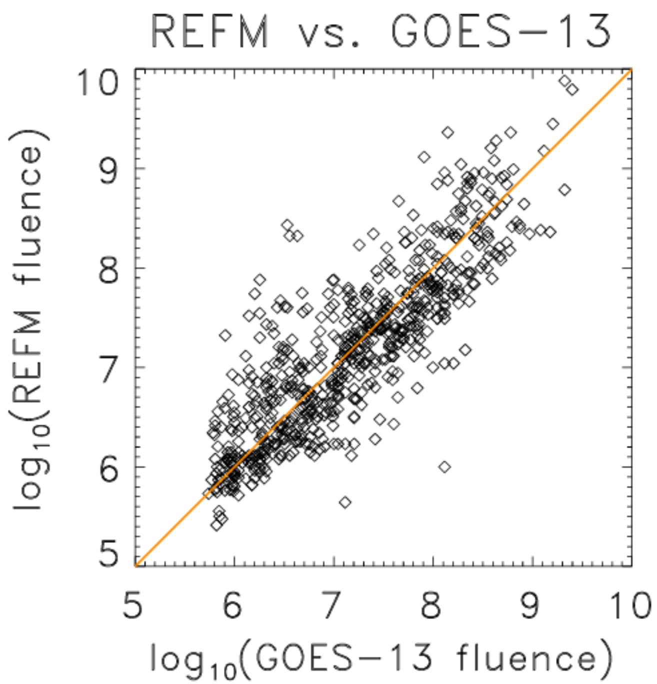


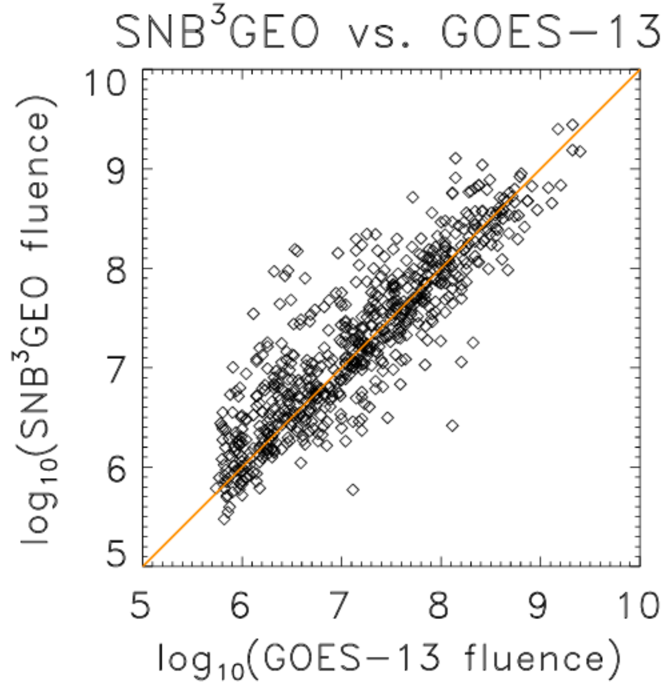
Electron Flux Model – SNB^{3}GEO

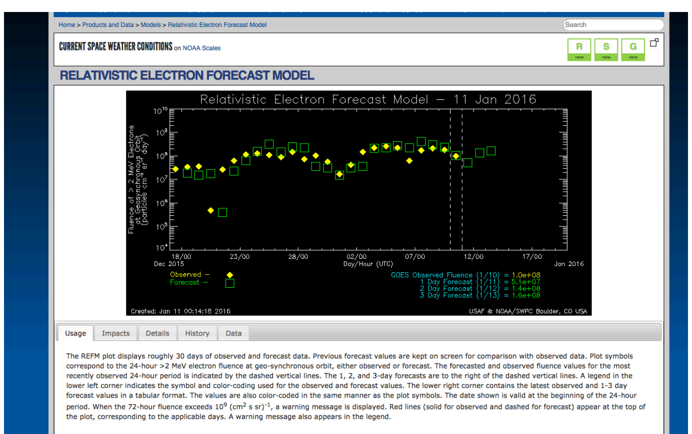


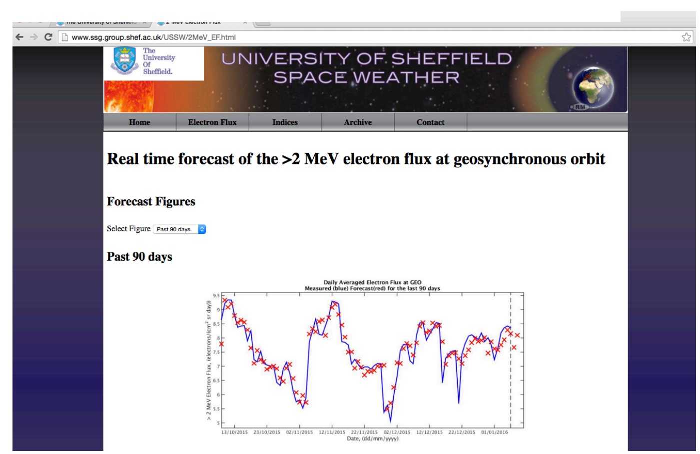
Electron flux – SNB^{3}GEO

NOAA-REFM vs. SNB^{3}GEO









Electron flux – SNB^3GEO

NOAA-REFM vs. SNB^3GEO

Balikhin et al. [2016], Space Weather

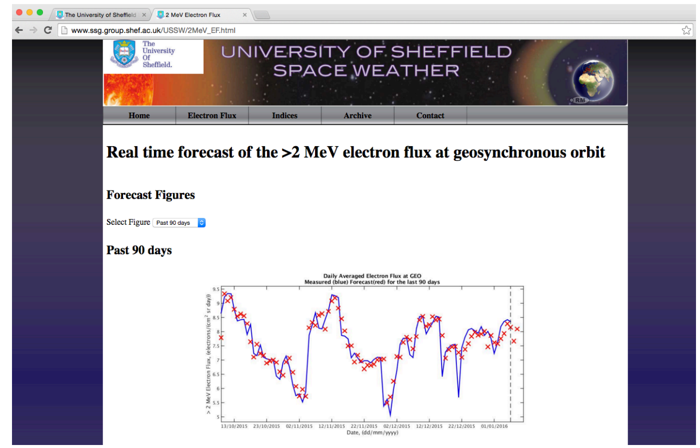
Fluxes

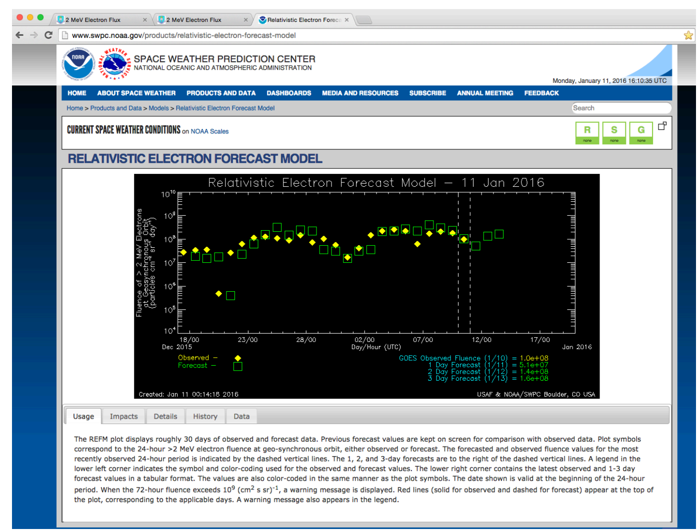
Model	Correlation	PE
REFM	0.73	-1.31
SNB^3GEO	0.82	0.63

$\log_{10}$(Fluxes)

Model	Correlation	PE
REFM	0.85	0.70
SNB^3GEO	0.89	0.77

March 2nd, 2012 - January 1st 2014.





Electron Flux Models - Performance

Heidke Skill score

Event Forecast	Event Observed		
	Yes	No	Marginal Total
Yes	a	b	a + b
No	c	d	c + d
Marginal Total	a + c	b + d	a + b + c + d = n

$$HSS = \frac{2(ad - bc)}{[(a+c)(c+d)+(a+b)(b+d)]}$$

Electron Flux Models - Performance

NOAA-REFM

Fluence ($cm^{-2}\ sr^{-1}\ d^{-1}$)	$>10^8$		$>10^{8.5}$		$>10^9$	
REFM HSS	0.666		0.482		0.437	
Observation	Yes	No	Yes	No	Yes	No
Forecast						
Yes	$x = 86$	$z = 22$	$x = 23$	$z = 22$	$x = 4$	$z = 7$
No	$y = 43$	$w = 510$	$y = 21$	$w = 595$	$y = 3$	$w = 647$

SNB^{3}GEO

Fluence ($cm^{-2}\ sr^{-1}\ d^{-1}$)	$>10^8$		$>10^{8.5}$		$>10^9$	
SNB^{3}GEO HSS	0.738		0.634		0.612	
Observation	Yes	No	Yes	No	Yes	No
Forecast						
Yes	$x = 106$	$z = 33$	$x = 31$	$z = 19$	$x = 4$	$z = 2$
No	$y = 23$	$w = 499$	$y = 13$	$w = 598$	$y = 3$	$w = 652$

Electron Flux Models: Low energies

A separate NARMAX model was developed for each of the 5 low energies (30-50 keV, 50-100 keV, 100-200 keV, 200-350 keV, 350-600 keV) using:

Output Data

GOES Electron Fluxes

Lags: 24 hours, 48 hours

Input Data

Solar wind Velocity V, Density n,

Pressure p, the Dst Index Dst,

and southward IMF B

Lags: 2 hours, 3 hours,…, 48 hours

Electron Flux Models: Low energies

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GOES Electron Fluxes

Lags: 24 hours, 48 hours

Input Data

Solar wind Velocity *V*, Density *n*, Pressure *p*, the Dst Index *Dst*, and southward IMF *B*

Lags: 2 hours, 3 hours,…, 48 hours

F is a fourth degree polynomial

$$\begin{aligned} J(t) = F[&J(t-24h), J(t-48h), \\ &v(t-2h), v(t-3h), \ldots, v(t-48h), \\ &n(t-2h), n(t-3h), \ldots, n(t-48h), \\ &p(t-2h), p(t-3h), \ldots, p(t-48h), \\ &Dst(t-2h), Dst(t-3h), \ldots, Dst(t-48h), \\ &B(t-2h), B(t-3h), \ldots, B(t-48h), \\ &e(t-24h), e(t-48h)] + e(t) \end{aligned}$$

Model	Forecast Horizon (hours)	PE (%)	CC (%)	Period
40-50 keV	10	66.9	82.0	01.03.2013-28.02.2015
50-100 keV	12	69.2	83.5	01.03.2013-28.02.2015
100-200 keV	16	73.2	85.6	01.03.2013-28.02.2015
200-350 keV	24	71.6	84.9	01.03.2013-28.02.2015
350-300 keV	24	73.6	85.9	01.03.2013-28.02.2015
> 800 keV	24	72.1	85.1	01.01.2011-28.02.2015
> 2MeV	24	82.3	90.9	01.0.12011-28.02.2015

Forecast Horizon of NARMAX models

The amount of time that the NARMAX model is able to forecast into the future is dependent on the minimum exogenous lag within the final NARMAX model.

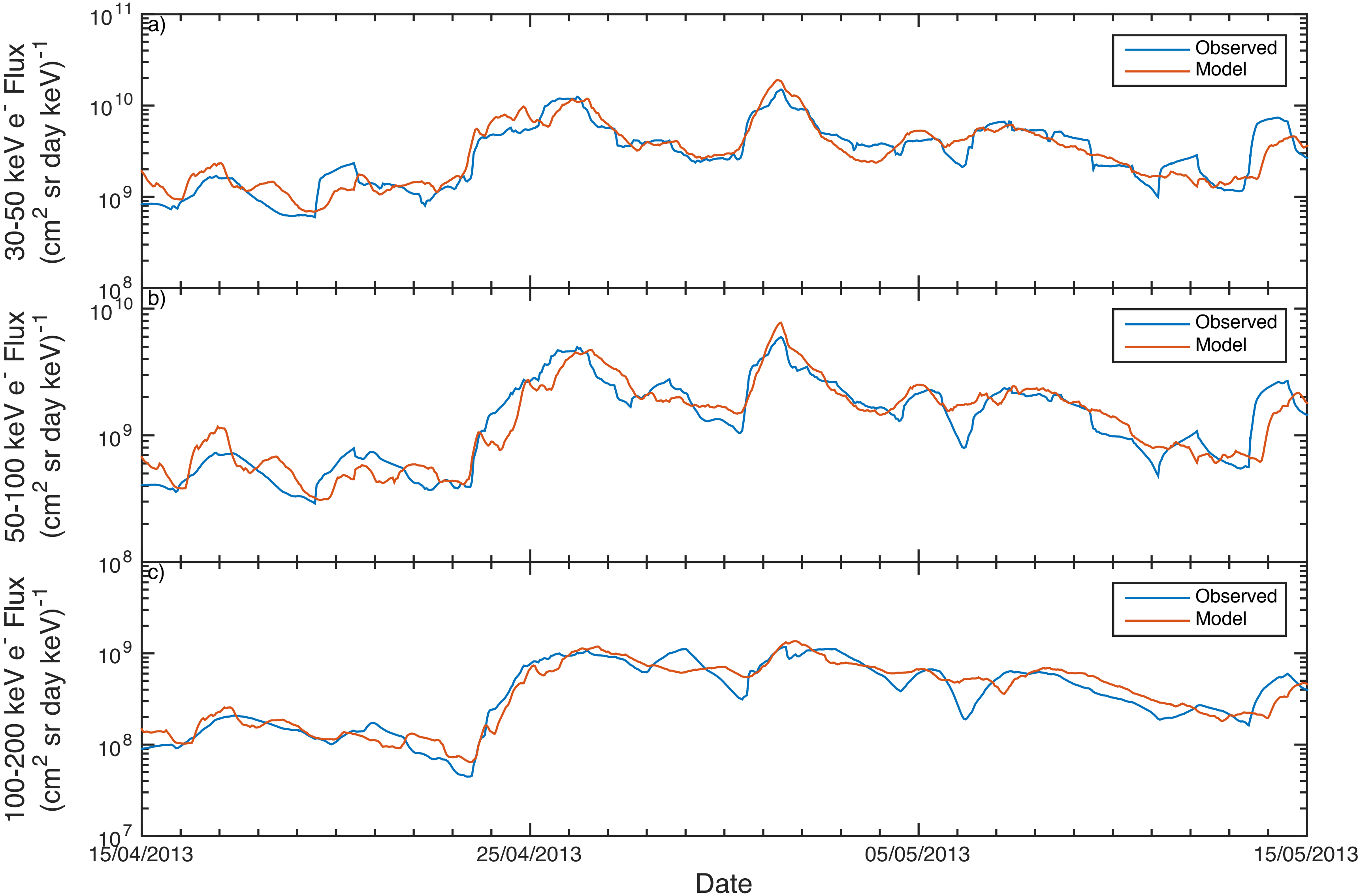
For example, if the minimum exogenous lag within the NARMAX model is a velocity value 10 hours ago

$$J(t) = aV(t-10) + \dots$$

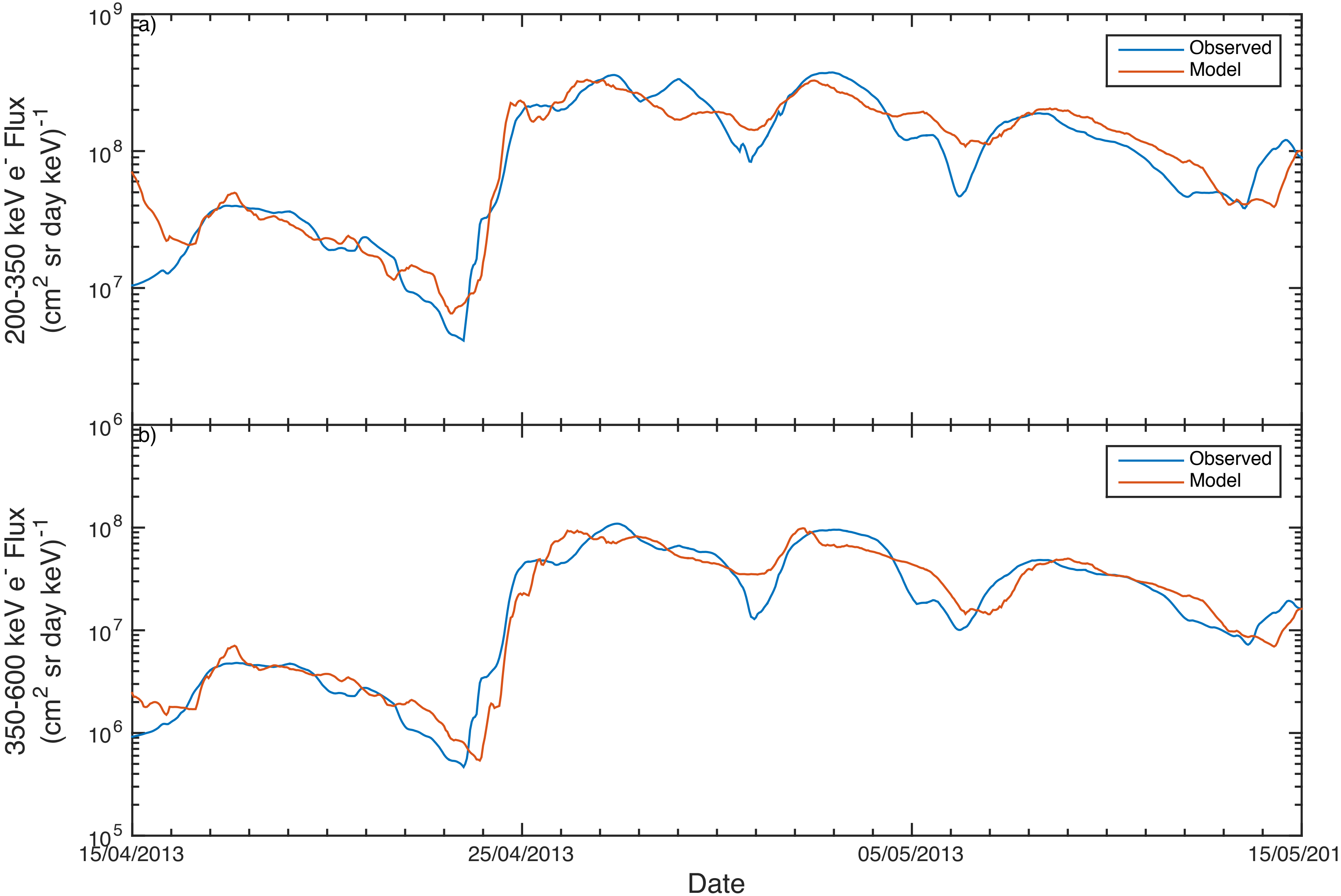
Where a is the coefficient, then if we know the velocity at the present time *t*, then we can calculate an estimate of the electron flux, *J*, at time *t*+10 hours (a 10 hour ahead forecast)

$$J(t+10) = aV(t) + \dots$$

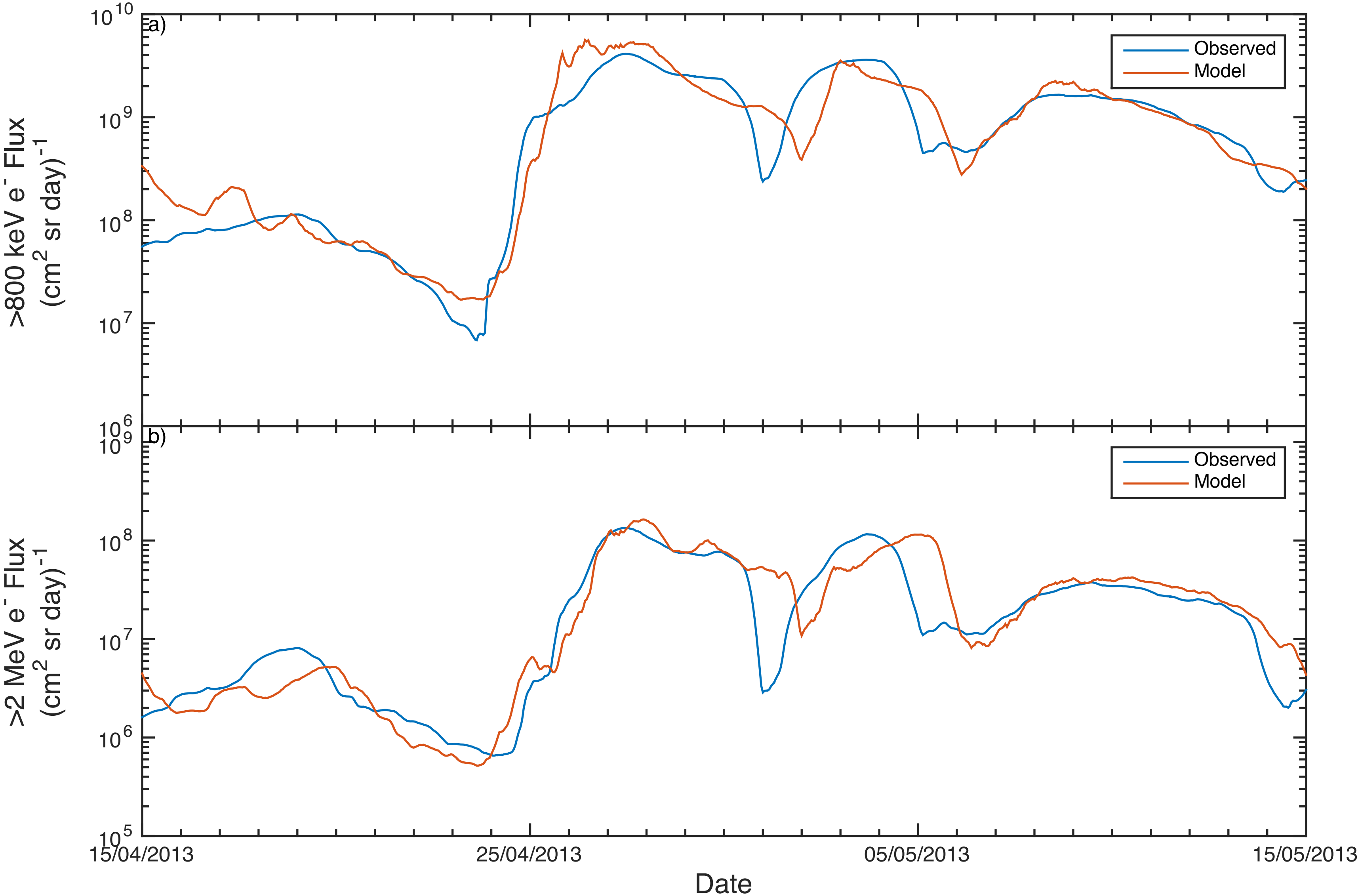
Model Performance Figures



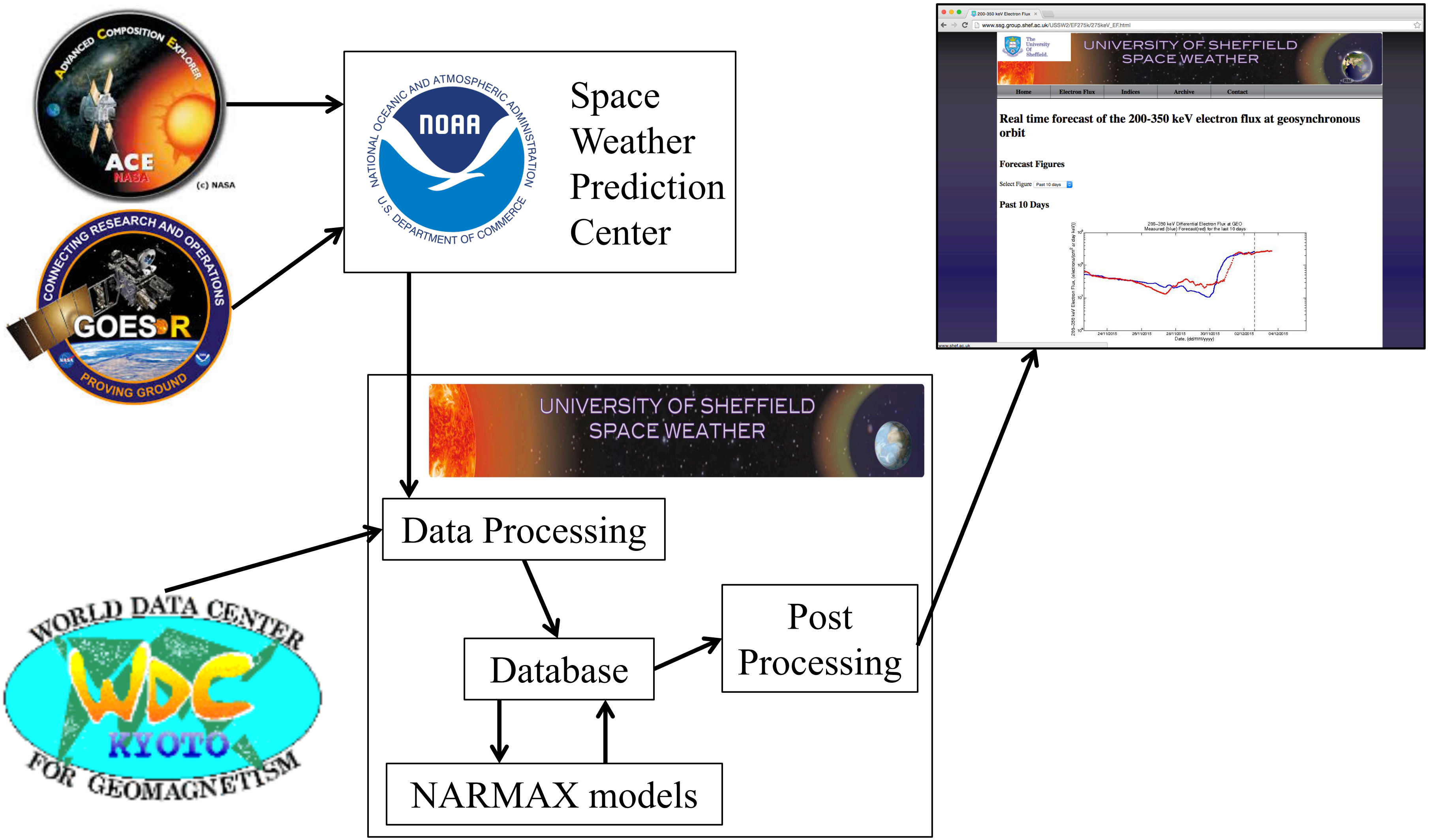
Model Performance Figures



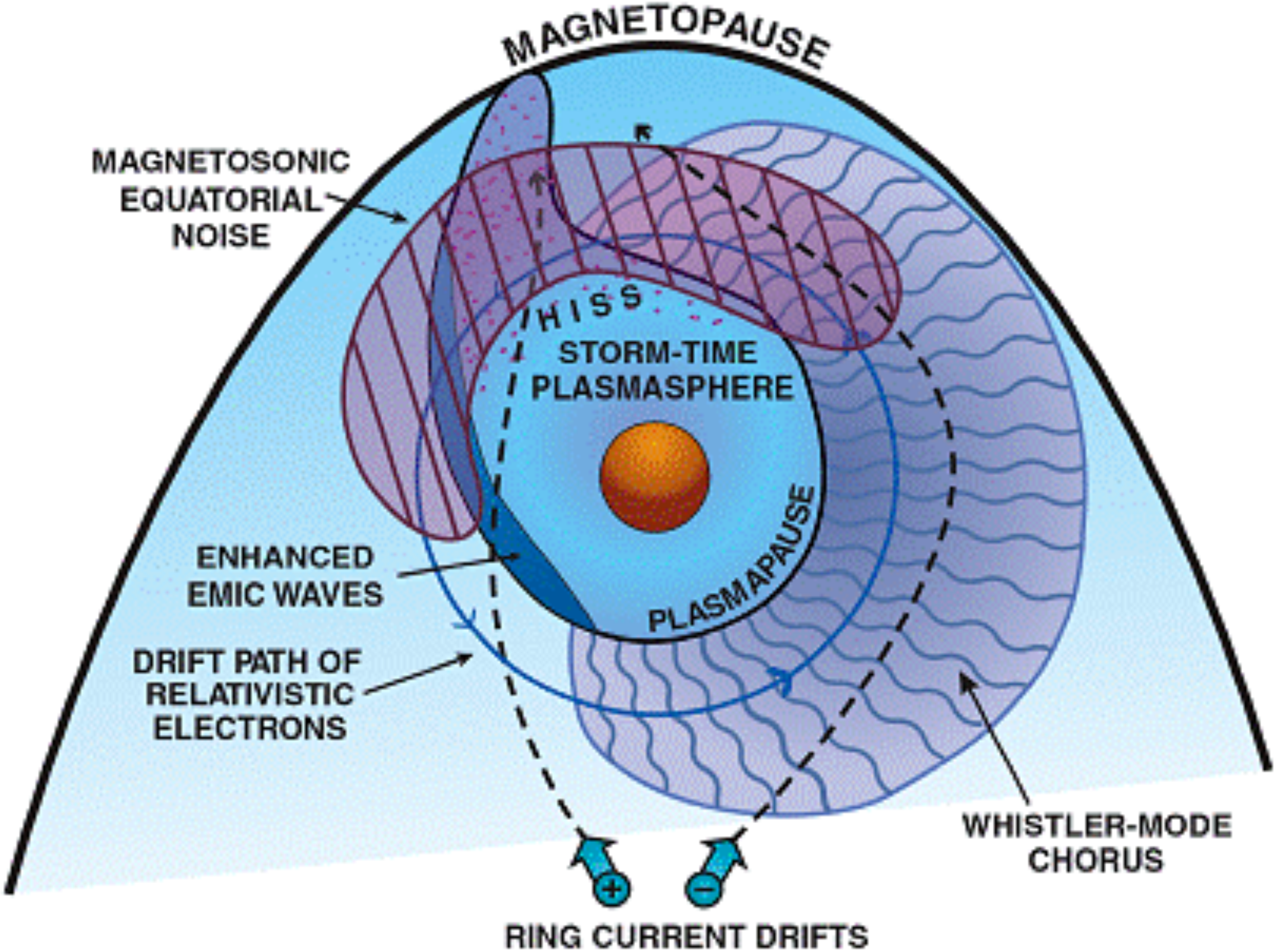
Model Performance Figures



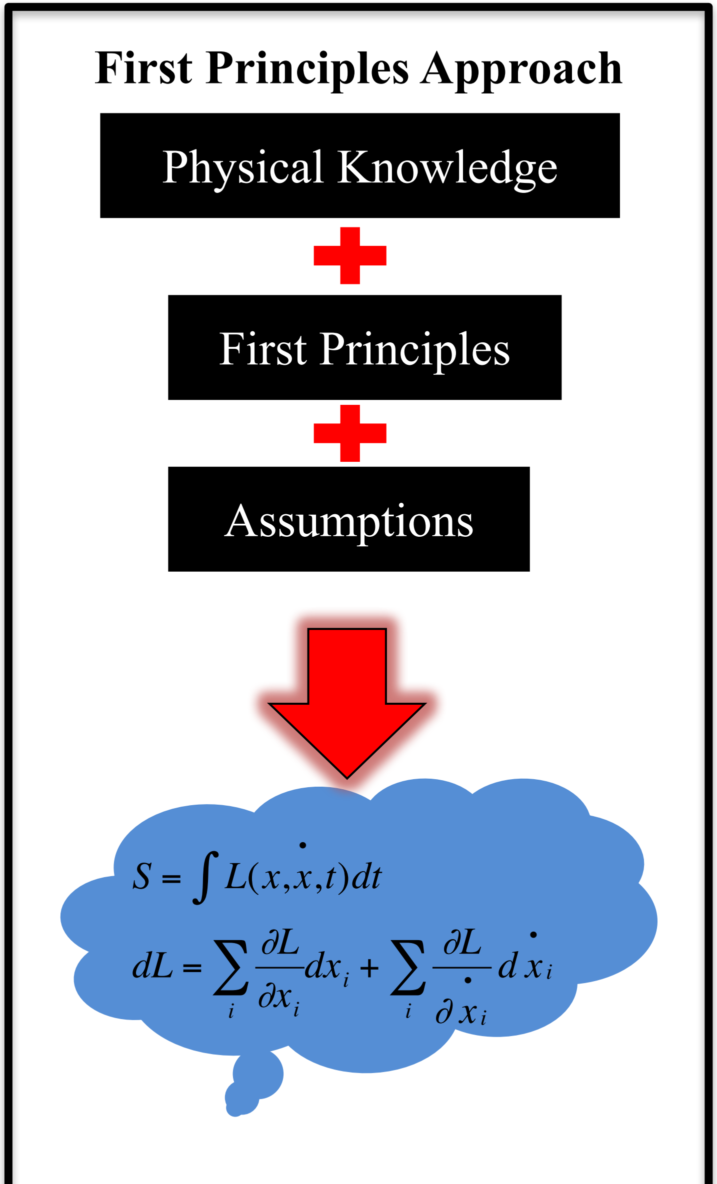
Real-time operation



Spatio-temporal modelling of radiation belt wave mode



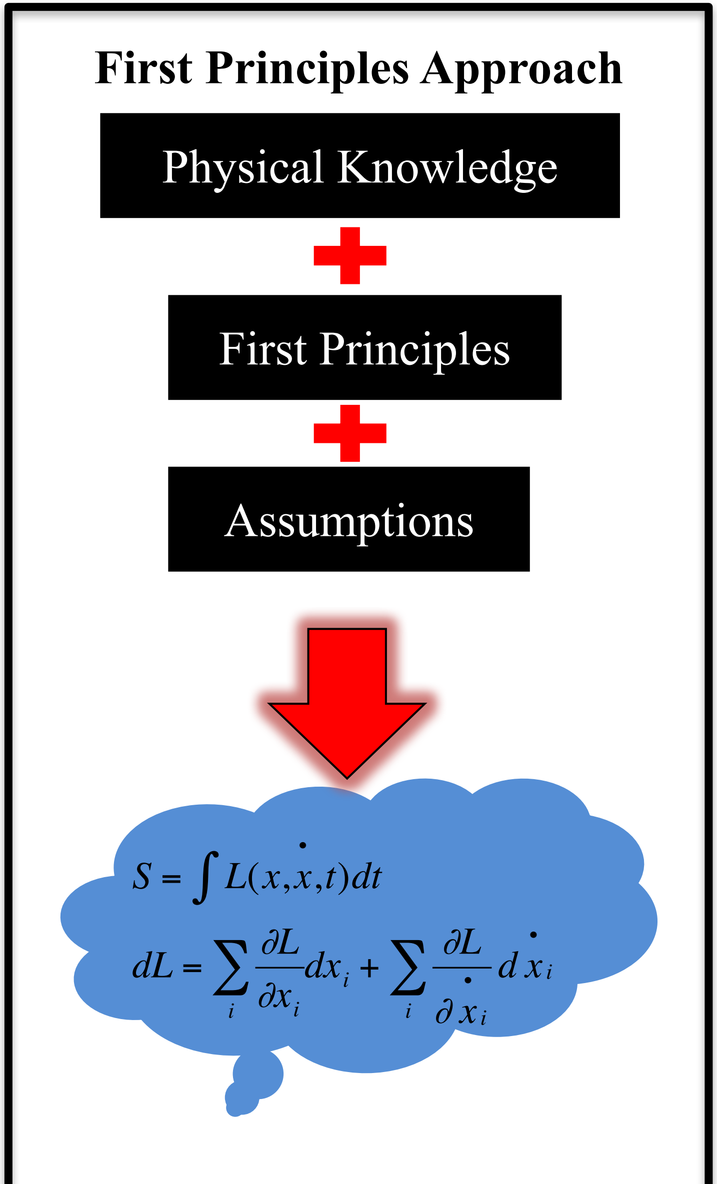
Physics Based Models



The evolution of the radiation belt electrons can be modelled by the bounce-averaged Fokker-Planck equation [Schulz and Lanzerotti, 1974]:

$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L^*}\bigg|_{\mu,J} \frac{1}{L^{*2}} D_{L^*L^*} \frac{\partial f}{\partial L^*}\bigg|_{\mu,J} + \frac{1}{p^2}\frac{\partial}{\partial p}\bigg|_{\alpha_0,\mathrm{L}}$$
$$\cdot\, p^2 \left(D_{pp} \frac{\partial}{\partial p}\bigg|_{\alpha_0,\mathrm{L}} f + D_{p\alpha_0} \frac{\partial}{\partial \alpha_0}\bigg|_{p,\mathrm{L}} f \right) + \frac{1}{T(\alpha_0)\sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0}\bigg|_{p,\mathrm{L}}$$
$$\cdot\, T(\alpha_0)\sin(2\alpha_0) \left(D_{\alpha_0\alpha_0} \frac{\partial}{\partial \alpha_0}\bigg|_{p,\mathrm{L}} f + D_{\alpha_0 p} \frac{\partial}{\partial p}\bigg|_{\alpha_0,\mathrm{L}} f \right) + \frac{f}{\tau},$$

Physics Based Models



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These models, such as Versatile Electron Radiation Belt (VERB) model employ numerical codes that involve finding solutions of the diffusion equations.

Diffusion Coefficients

Many approaches have been developed to calculate the diffusion coefficients, all of which require models of various waves.

For example, the VERB code employs statistical wave models for Lower Band Chorus (LBC), Hiss and Equatorial MagnetoSonic (EMS) waves.

Currently, the statistical models of the waves distributions employ wave measurements on various spacecraft, which are parameterized by the location of observations and current values for geomagnetic indices neglecting solar wind measurements and geomagnetic evolution.

Aims

Work Package 4 of PROGRESS aims to determine the influential parameters (solar wind and geomagnetic indices) that control the wave amplitude distribution at particular locations and then redevelop the statistical wave models

Diffusion coefficients

Statistical wave models

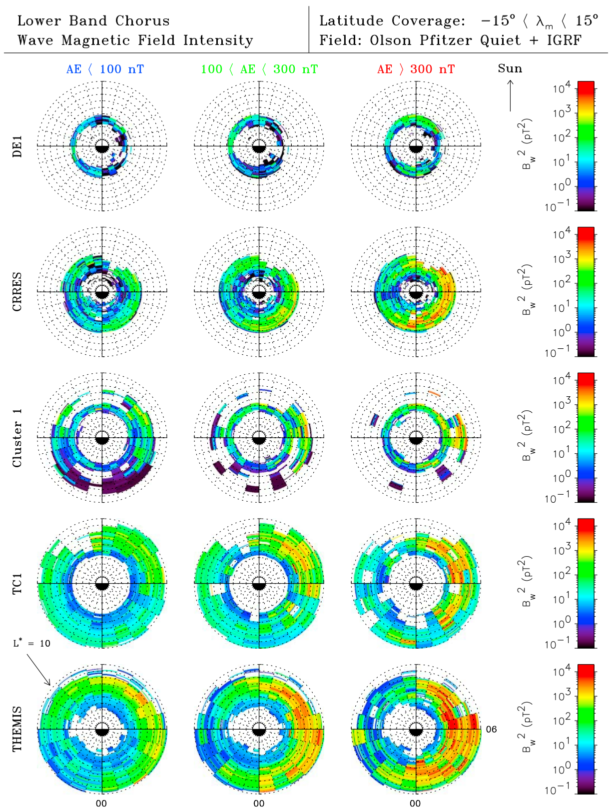
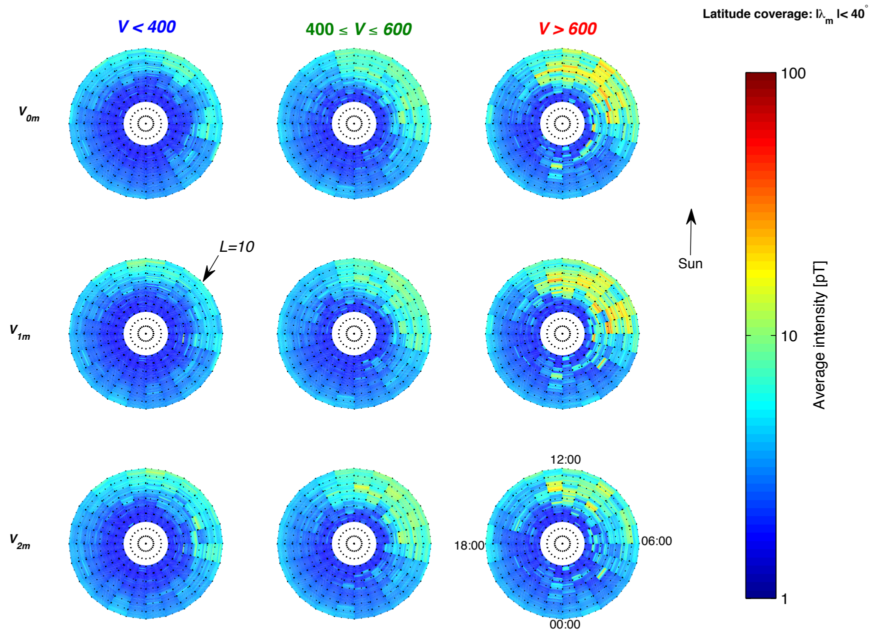


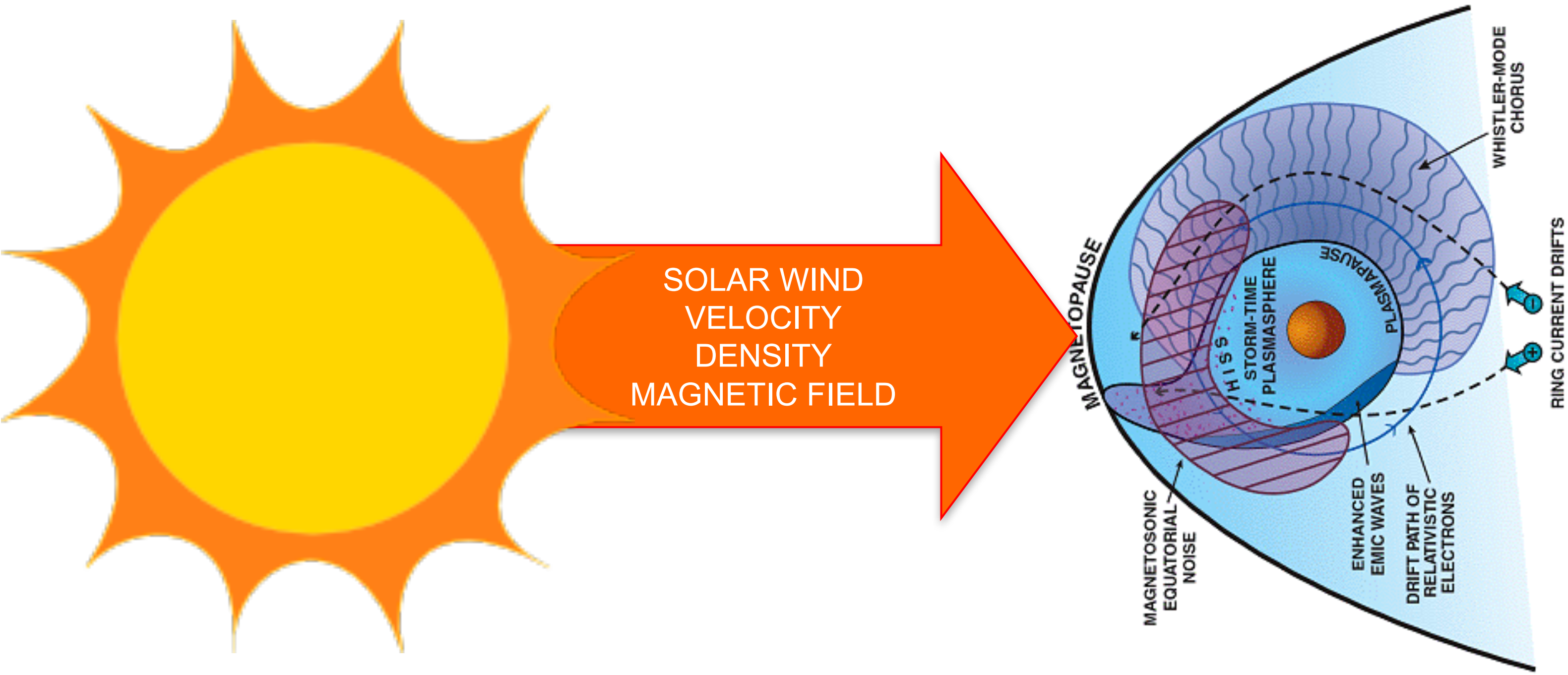
Figure 2. Equatorial wave intensity of lower band chorus as a function of L^*, MLT and geomagnetic activity for each of the five satellites.

Meredith et al., JGR, 2012



Aryan et al., JGR, 2014

How to identify wave control parameters



What parameters influence the waves in the radiations belts?

How to identify these parameters?

Correlation

A simple quadratic system

$$y(t) = x^2(t-1) + \mathrm{e(t)}$$

Where the output *y* at time *t* is a function of zero mean signal *x* and noise *e*

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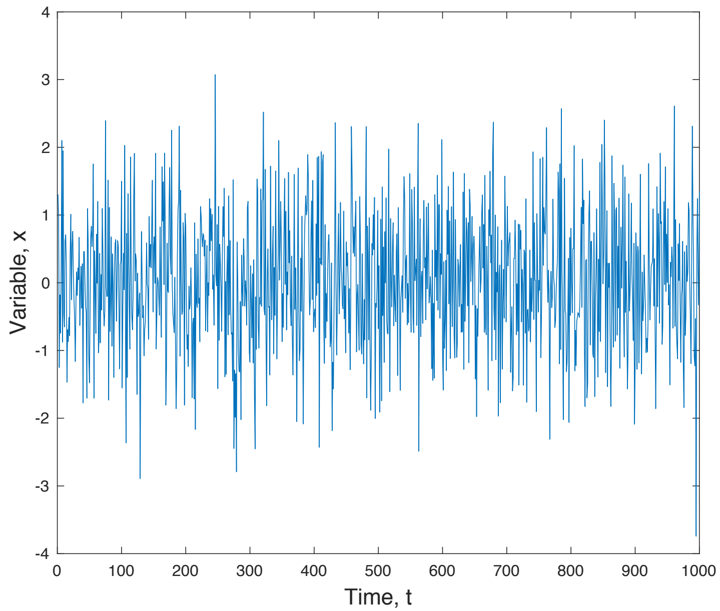

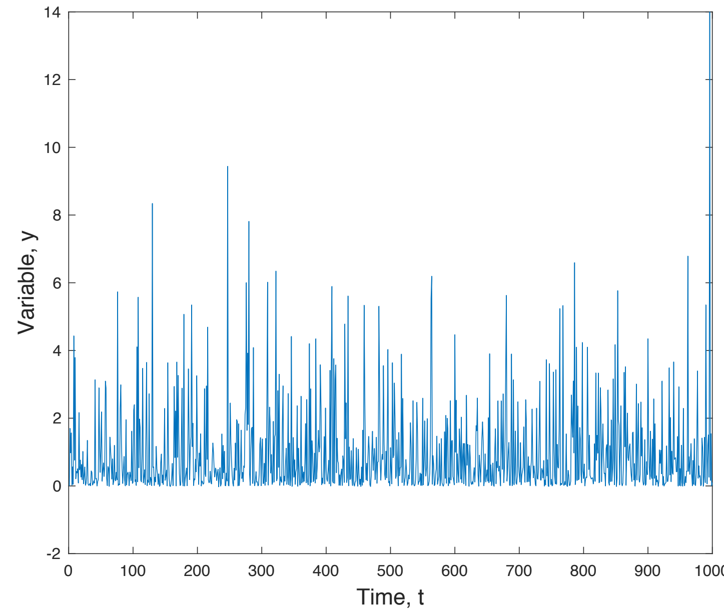

Correlation

A simple quadratic system

$$y(t) = x^2(t-1) + \mathrm{e(t)}$$

Where the output *y* at time *t* is a function of zero mean signal *x* and noise *e*

Calculate the correlation function:

$$\phi_{xy}(\tau) = \frac{\sum_{t=1}^{N} \left[\left(y(t-\tau) - \bar{y} \right) \left(x(t) - \bar{x} \right) \right]}{\sqrt{\sum_{t=1}^{N} \left[\left(y(t) - \bar{y} \right)^2 \right] \sum_{t=1}^{N} \left[\left(x(t) - \bar{x} \right)^2 \right]}}$$

Correlation

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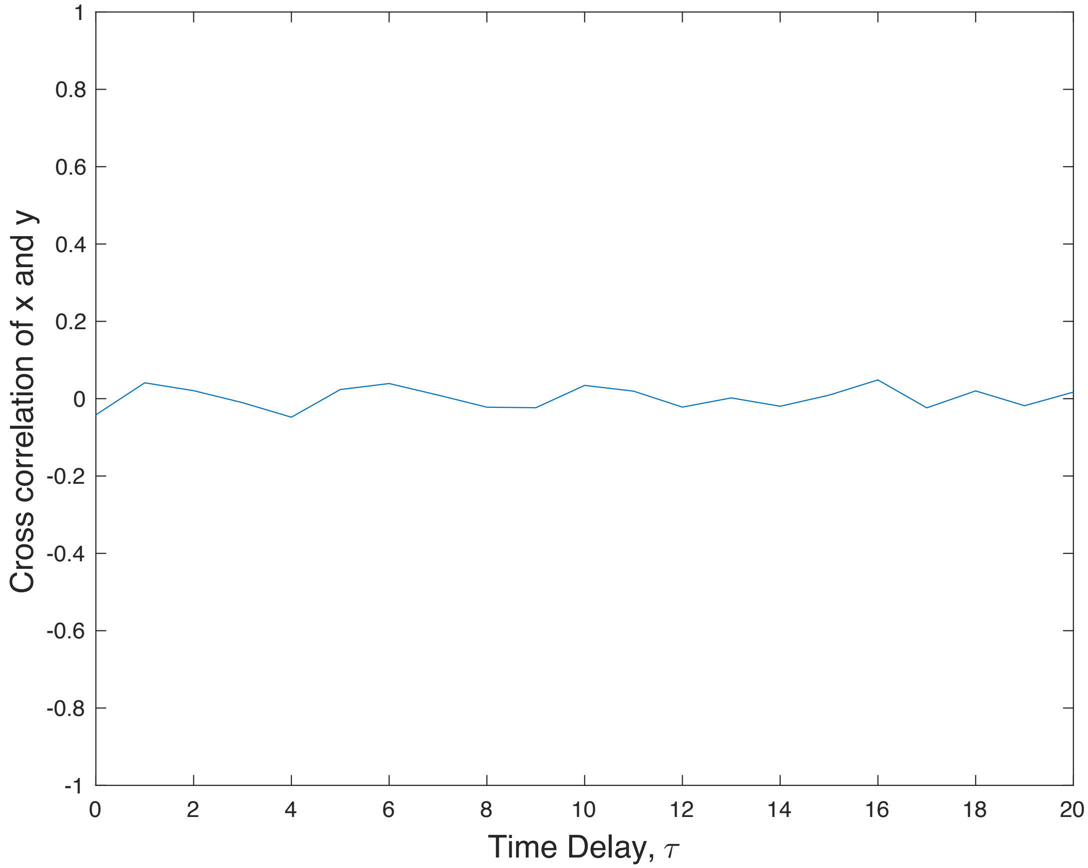

Correlation

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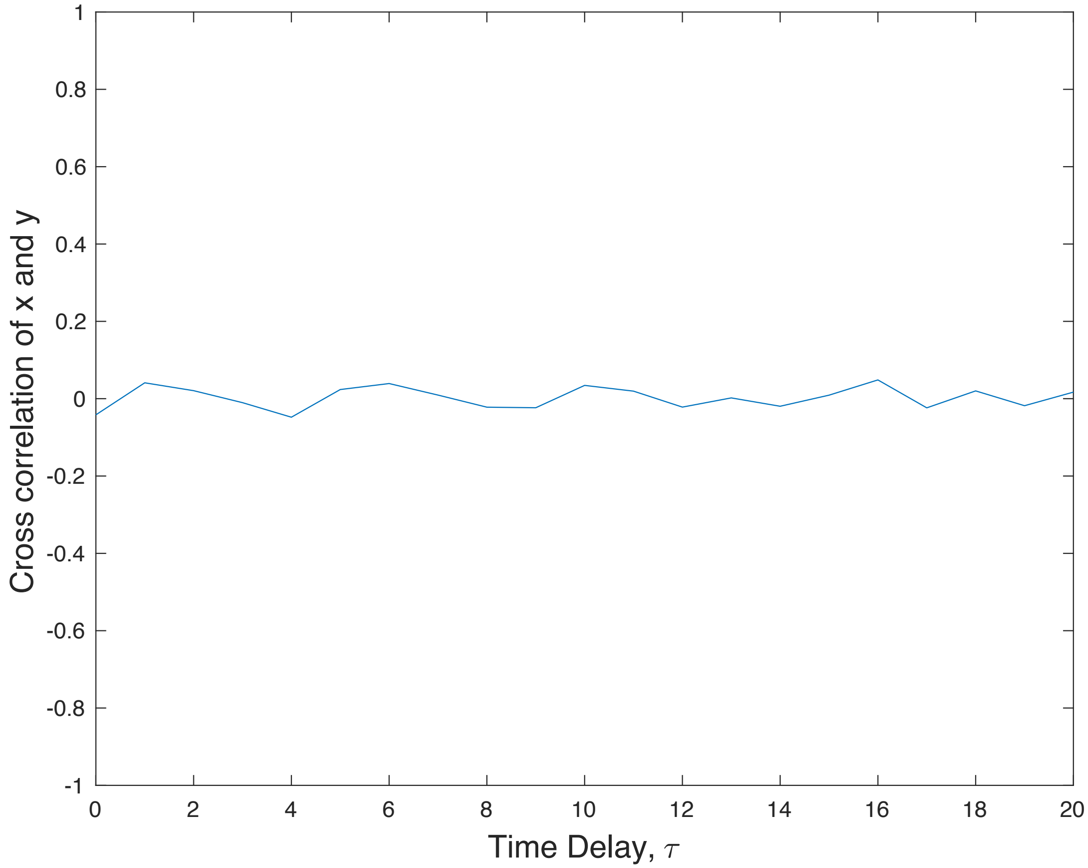
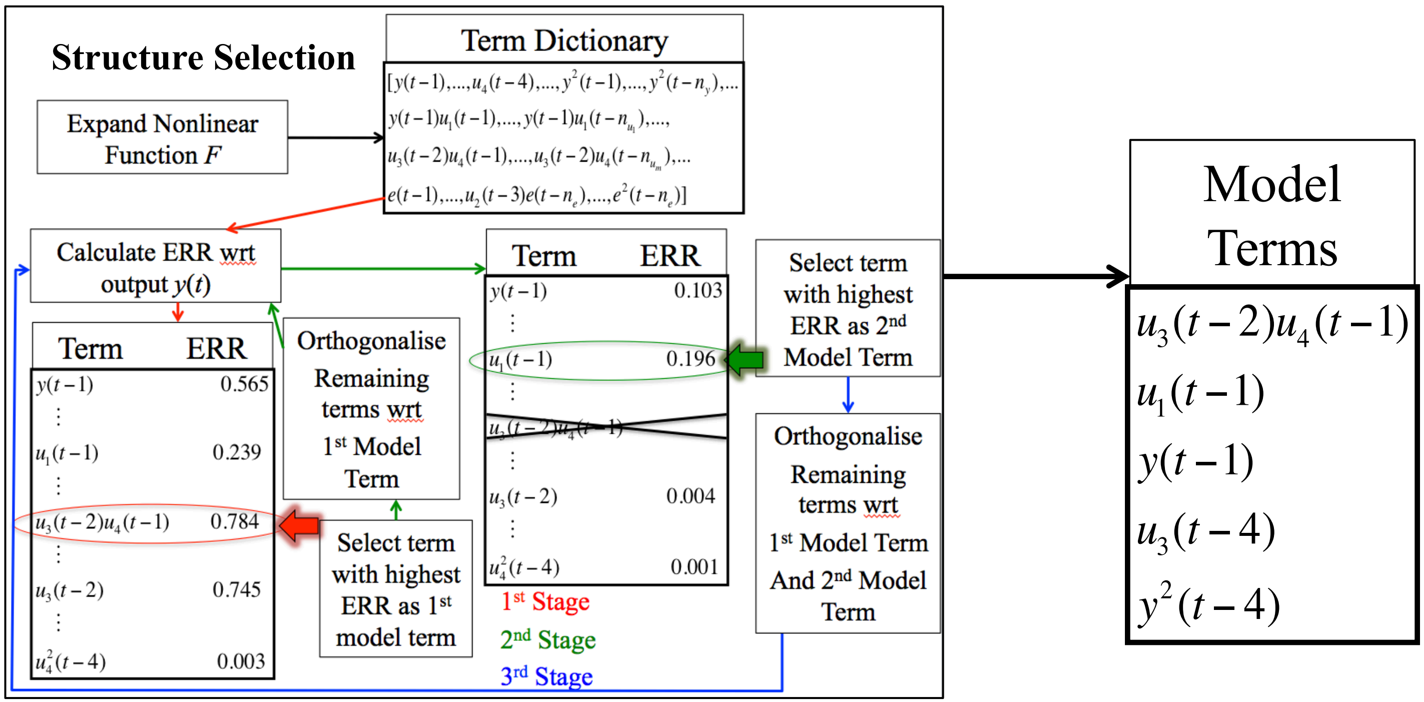


Correlation between x and y is ~0 for all delays even though x is the input to the system

Therefore, the correlation function can be misleading when applying it to nonlinear systems!

NARMAX FROLS ERR

Better to use techniques that are able to account for nonlinear systems, such as NARMAX FROLS ERR



NARMAX FROLS ERR

A simple quadratic system

$$y(t) = x^2(t-1) + \mathrm{e(t)}$$

Where the output *y* at time *t* is a function of zero mean signal *x* and noise *e*

NARX model:

$$y(t) = F[y(t-1), y(t-2), x(t-1), x(t-2), v(t-3), v(t-1), v(t-2), v(t-3)]$$

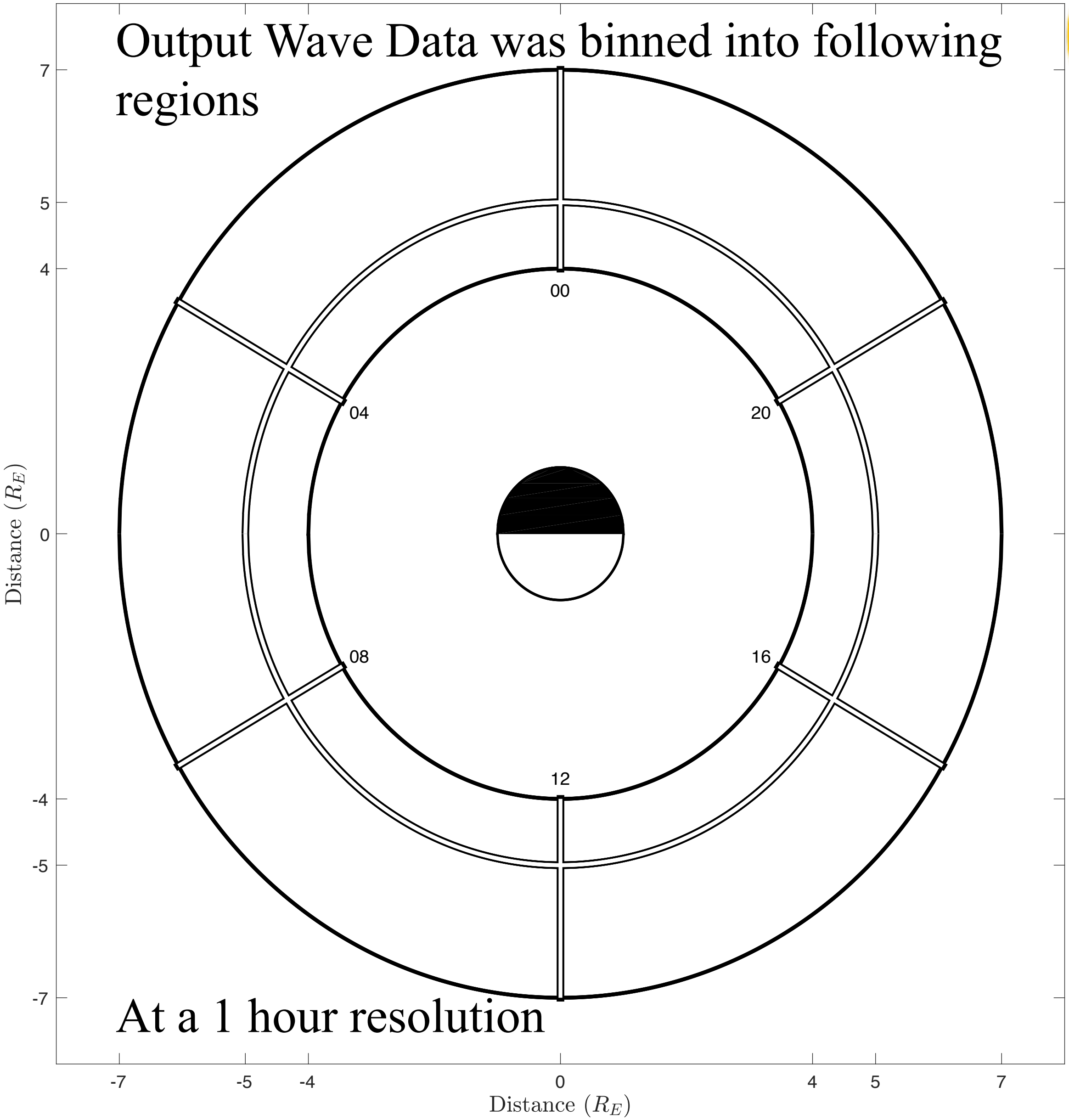
F as a third degree polynomial

Where *v* was a random variable

TERM	ERR (%)
$x^2(t\text{-}1)$	98.6

Wave data

Output Wave Data was binned into following regions



At a 1 hour resolution

ERR analysis of radiation belt waves

Output Data

Wave intensity, B_w , for each MLT, L bin

From THEMIS, Cluster and Double Star

Input Data

Solar wind Velocity V,

Density n,

Pressure p,

and IMF factor B_B

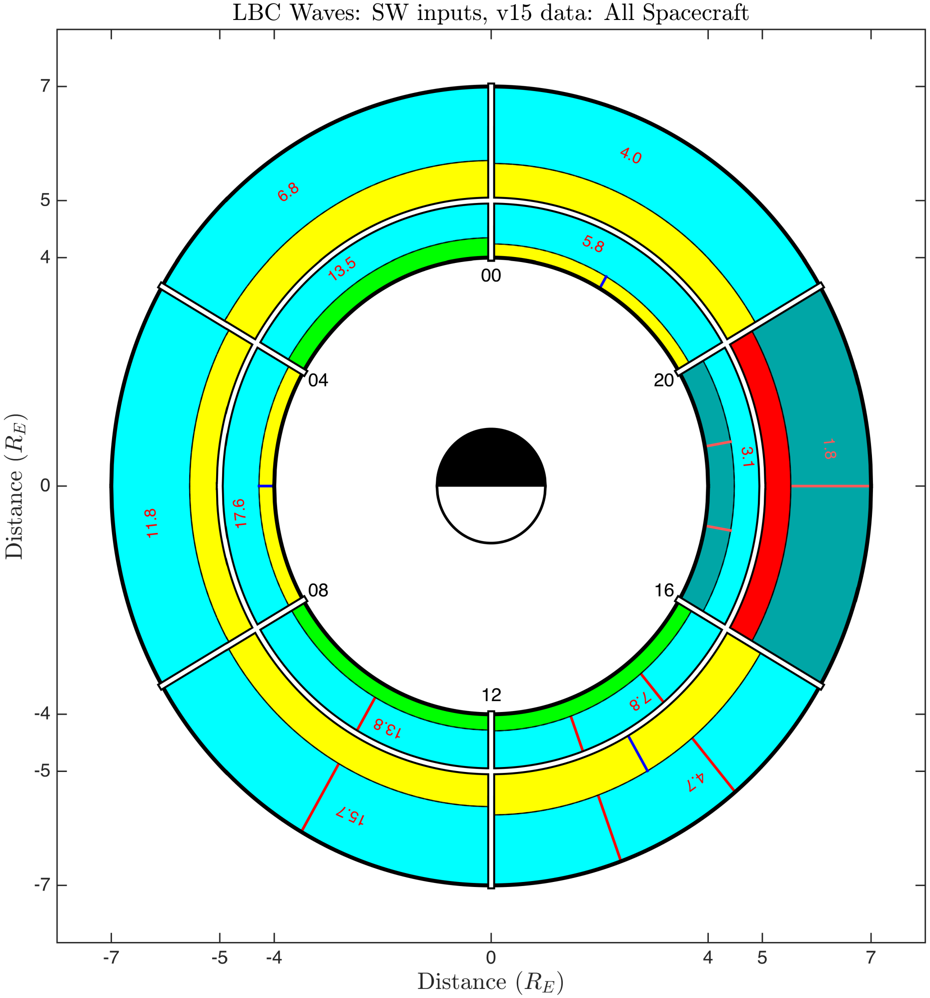
Lags: 0 hours, 2 hours, 4 hours,…, 20 hours

$$\begin{aligned} B_w(L, MLT, t) = F[&V(t-0), V(t-2), ..., V(t-20), \\ &n(t-0), n(t-2), ..., n(t-20), \\ &p(t-0), p(t-2), ..., p(t-20), \\ &B_B(t-0), B_B(t-2), ..., B_B(t-20)] \end{aligned}$$

N.B. F contains no autoregressive or moving average terms as wave data is too sparse

ERR analysis of radiation belt waves: LBC waves

Linear *F*

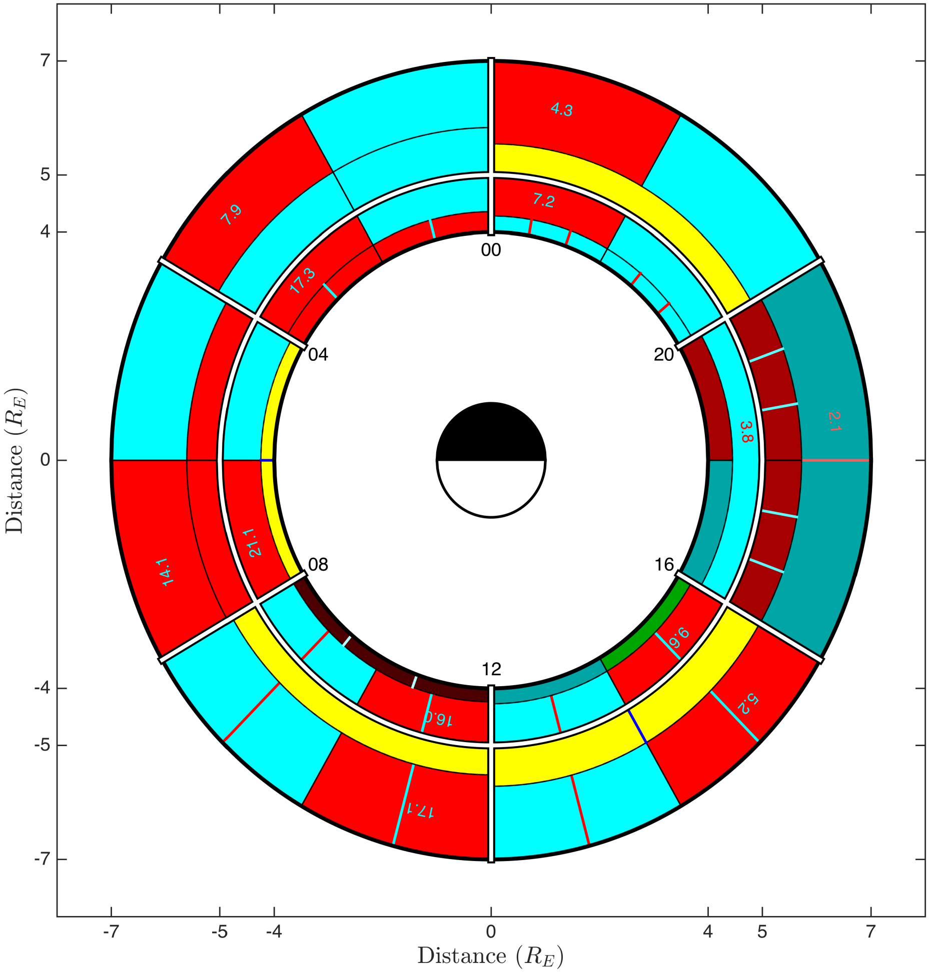


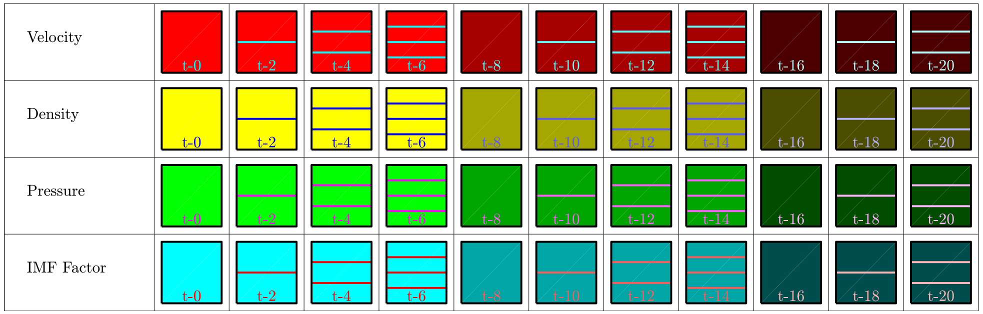
Velocity	t-0	t-2	t-4	t-6	t-8	t-10	t-12	t-14	t-16	t-18	t-20
Density	t-0	t-2	t-4	t-6	t-8	t-10	t-12	t-14	t-16	t-18	t-20
Pressure	t-0	t-2	t-4	t-6	t-8	t-10	t-12	t-14	t-16	t-18	t-20
IMF Factor	t-0	t-2	t-4	t-6	t-8	t-10	t-12	t-14	t-16	t-18	t-20

ERR analysis of radiation belt waves: LBC waves

Quadratic *F*

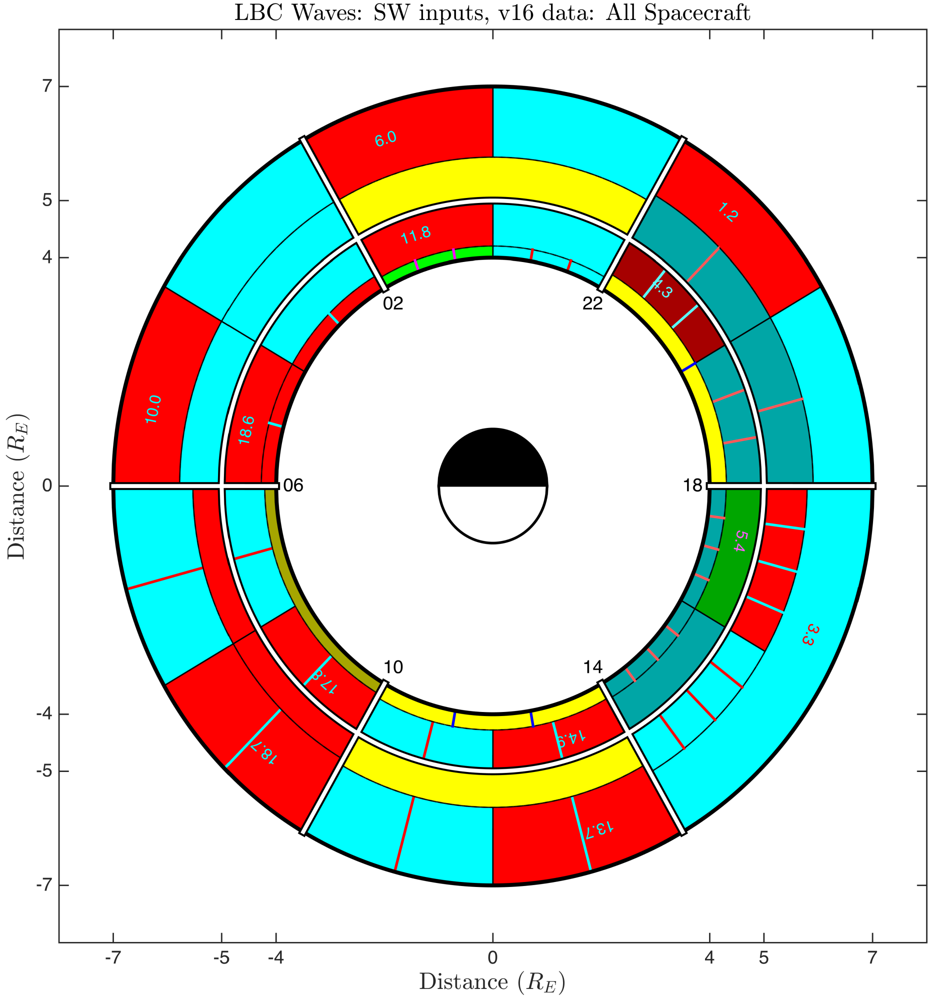
LBC Waves: SW inputs, v15 data: All Spacecraft

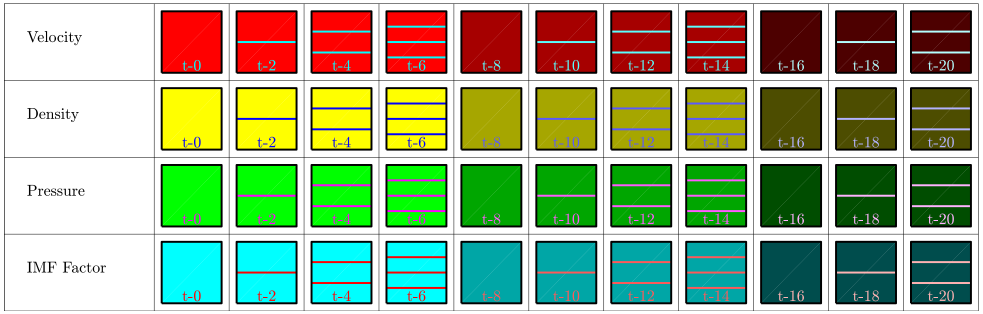




ERR analysis of radiation belt waves: LBC waves

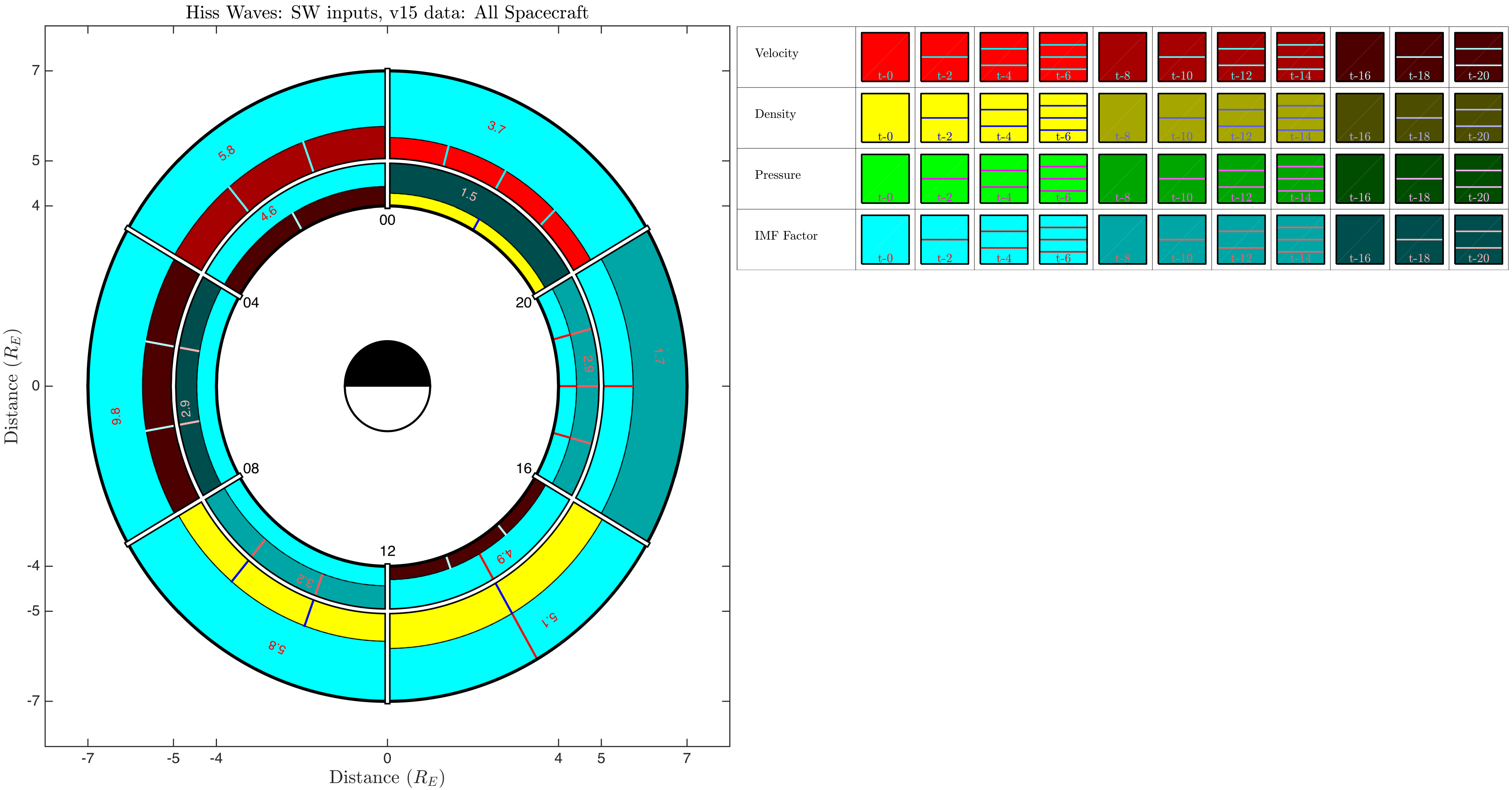
Quadratic *F* Rotated bins





ERR analysis of radiation belt waves: Hiss waves

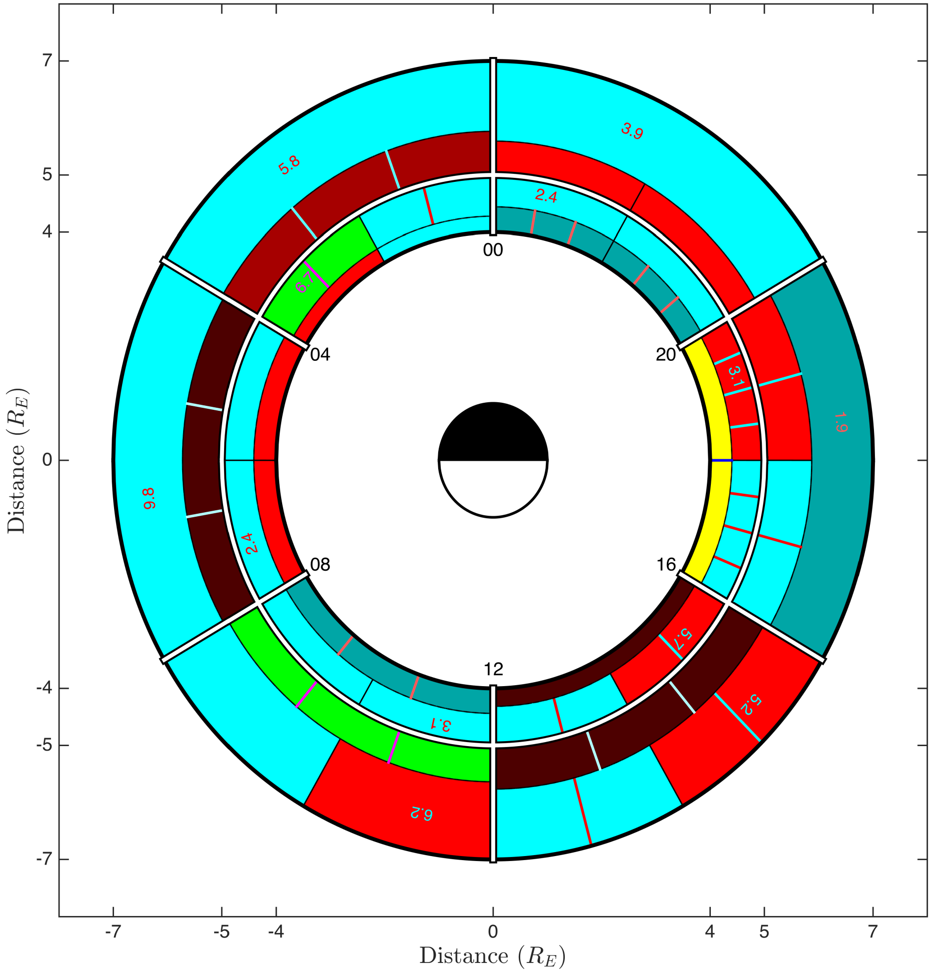
Linear *F*



ERR analysis of radiation belt waves: Hiss waves

Quadratic *F*

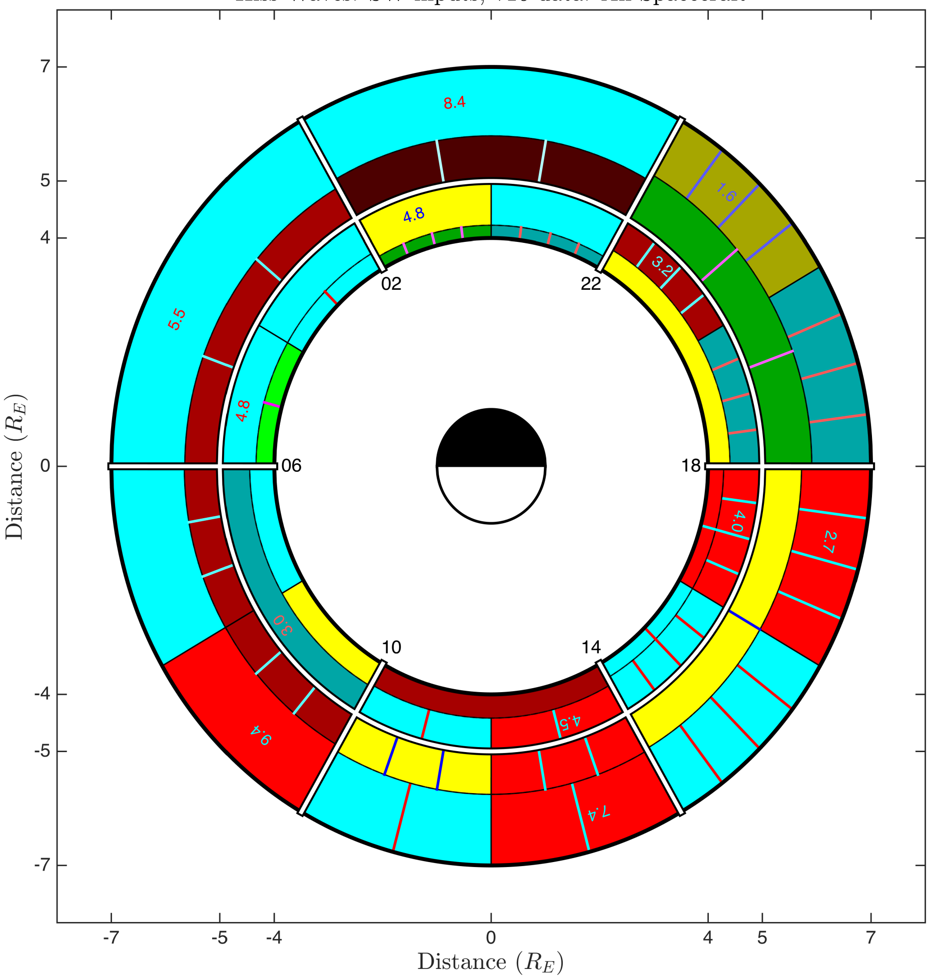
Hiss Waves: SW inputs, v15 data: All Spacecraft

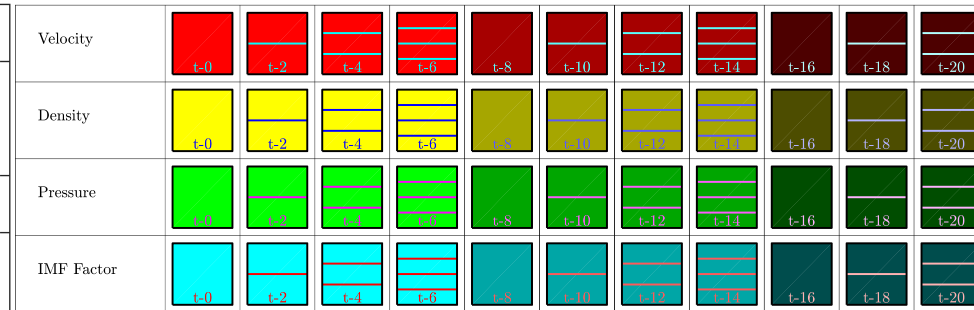


Velocity	t-0	t-2	t-4	t-6	t-8	t-10	t-12	t-14	t-16	t-18	t-20
Density	t-0	t-2	t-4	t-6	t-8	t-10	t-12	t-14	t-16	t-18	t-20
Pressure	t-0	t-2	t-4	t-6	t-8	t-10	t-12	t-14	t-16	t-18	t-20
IMF Factor	t-0	t-2	t-4	t-6	t-8	t-10	t-12	t-14	t-16	t-18	t-20

ERR analysis of radiation belt waves: Hiss waves

Quadratic *F* Rotated bins

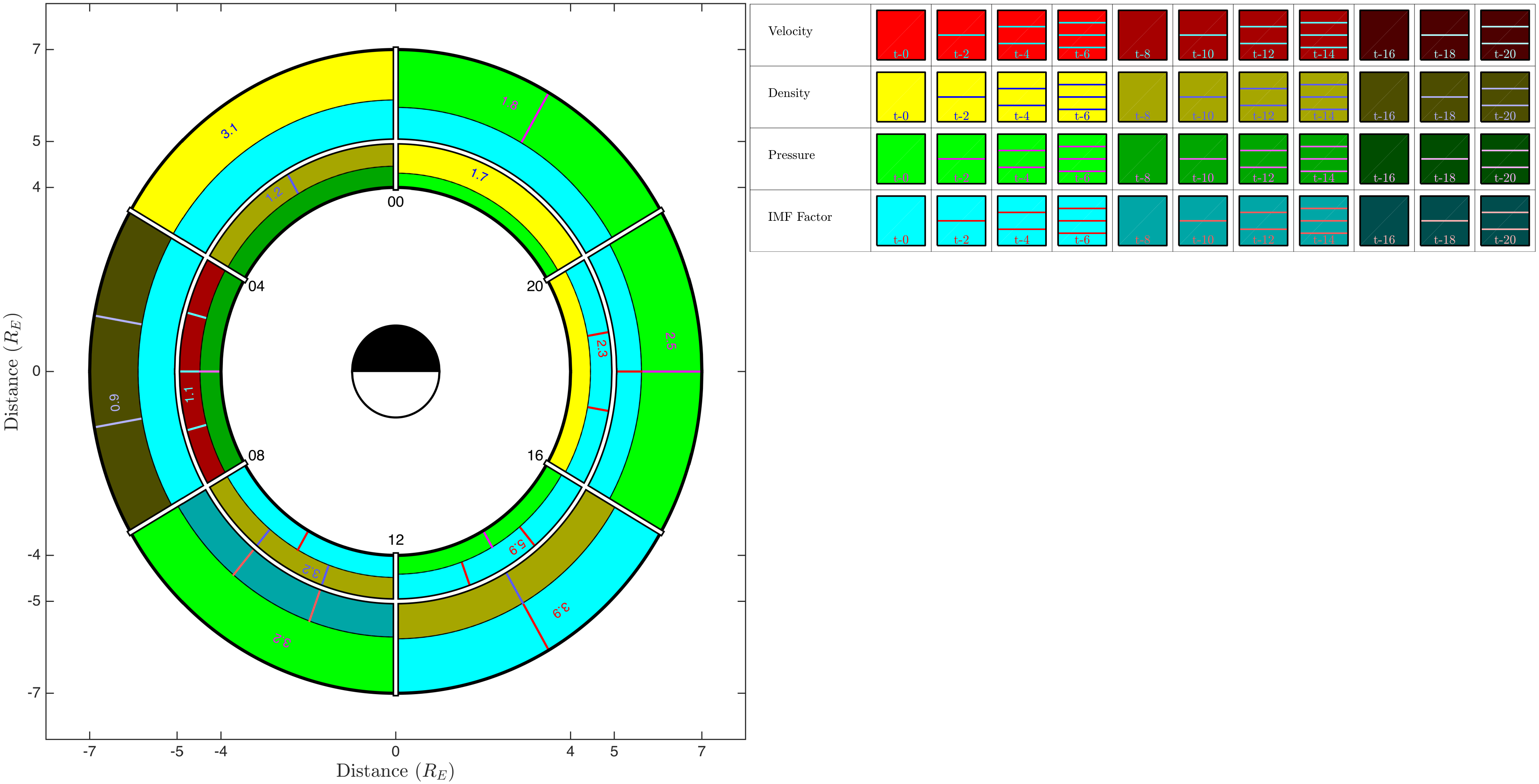




ERR analysis of radiation belt waves: EMS waves

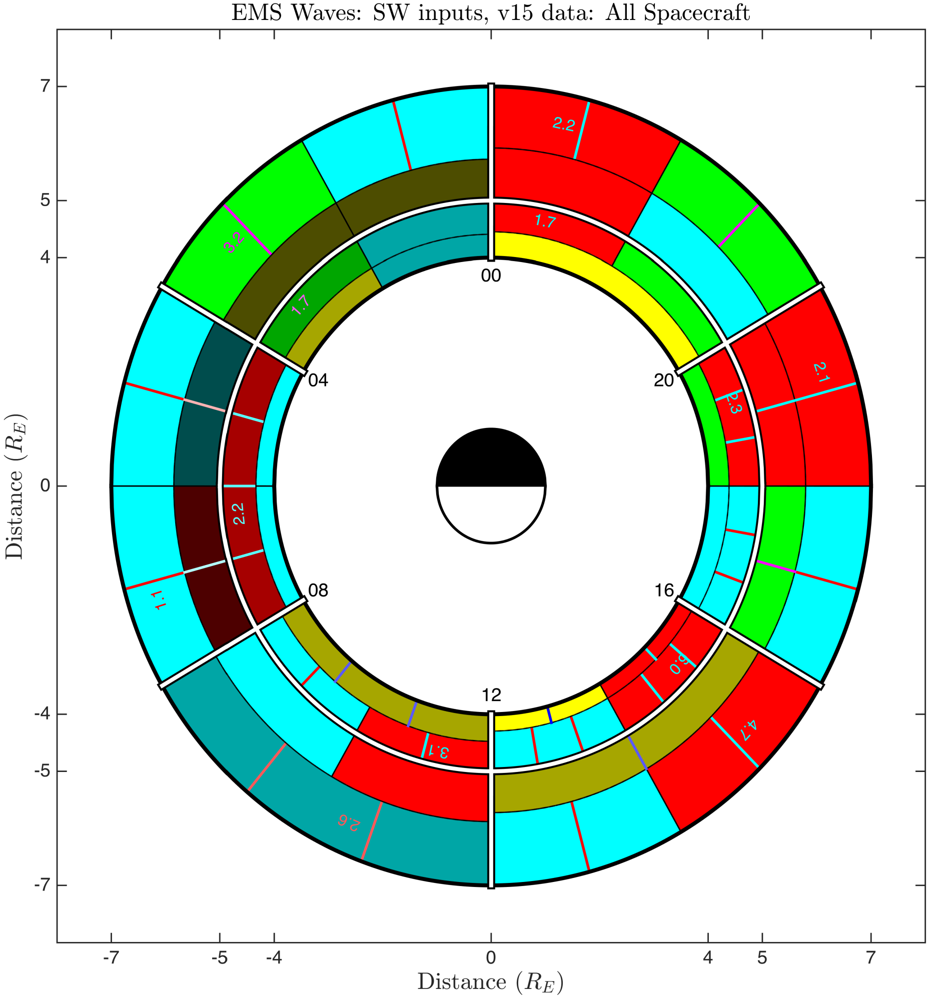
Linear *F*

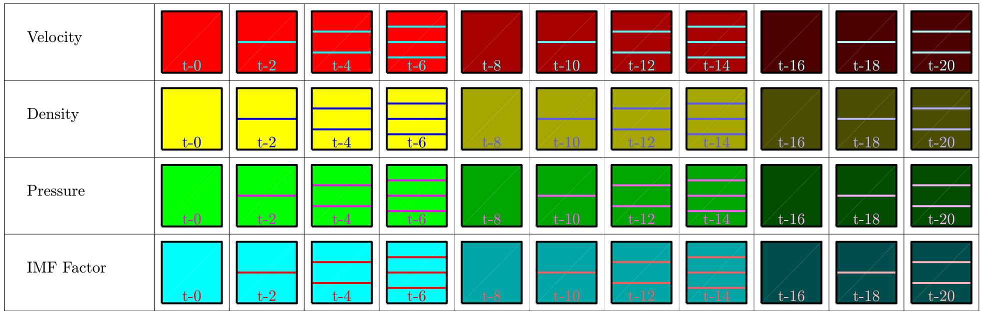
EMS Waves: SW inputs, v15 data: All Spacecraft



ERR analysis of radiation belt waves: EMS waves

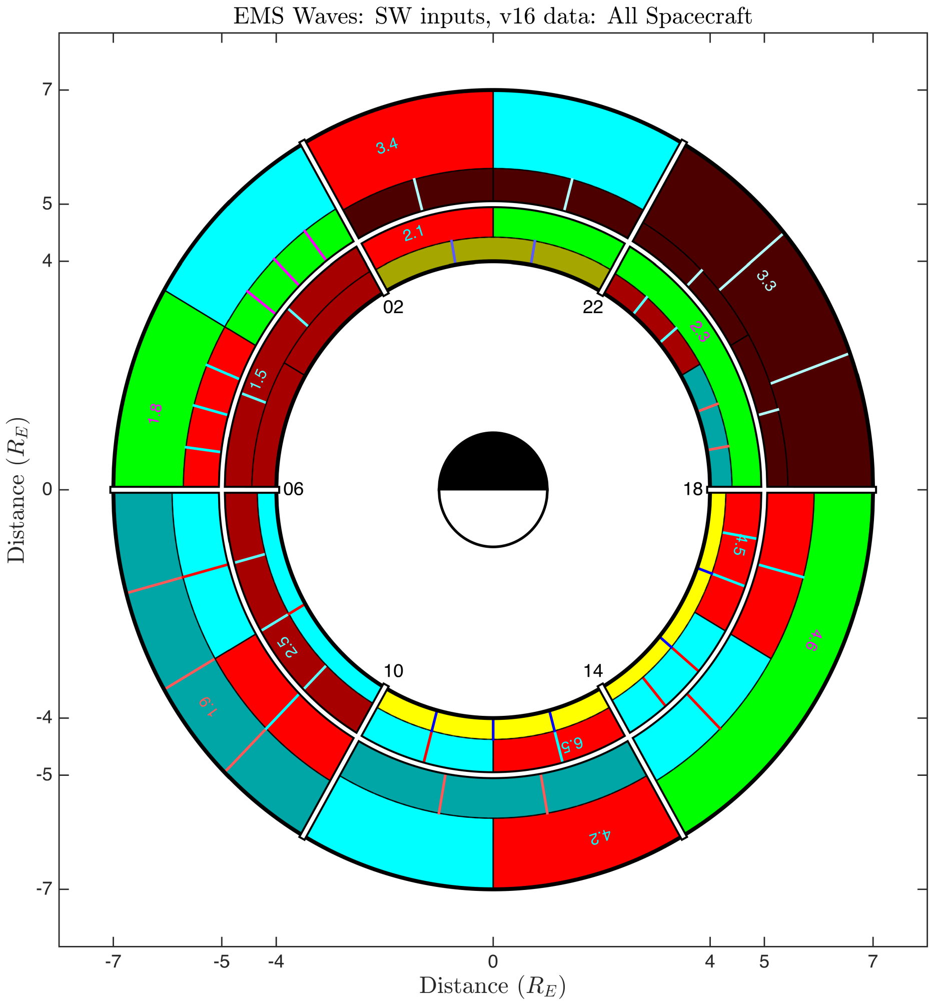
Quadratic *F*

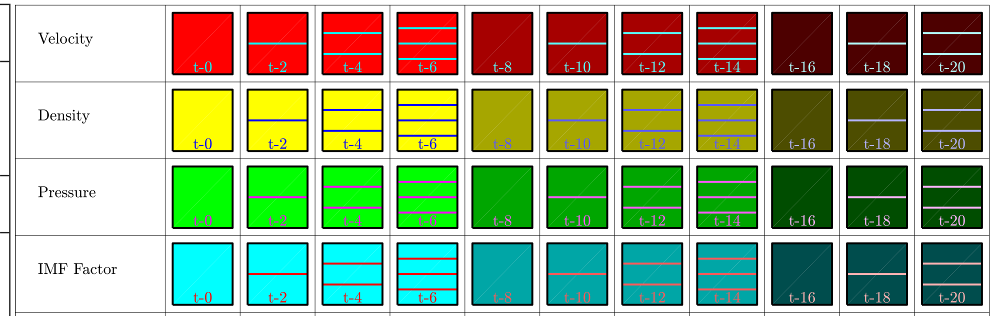




ERR analysis of radiation belt waves: EMS waves

Quadratic *F* Rotated bins





ERR analysis of radiation belt waves

Output Data

Wave intensity, B_w , for each MLT, L bin

From THEMIS, Cluster and Double Star

Input Data

Solar wind Velocity V,

Density n,

Pressure p,

IMF factor B_B ,

Dst index Dst,

And AE index AE.

Lags: 2 hours, 4 hours,…, 20 hours

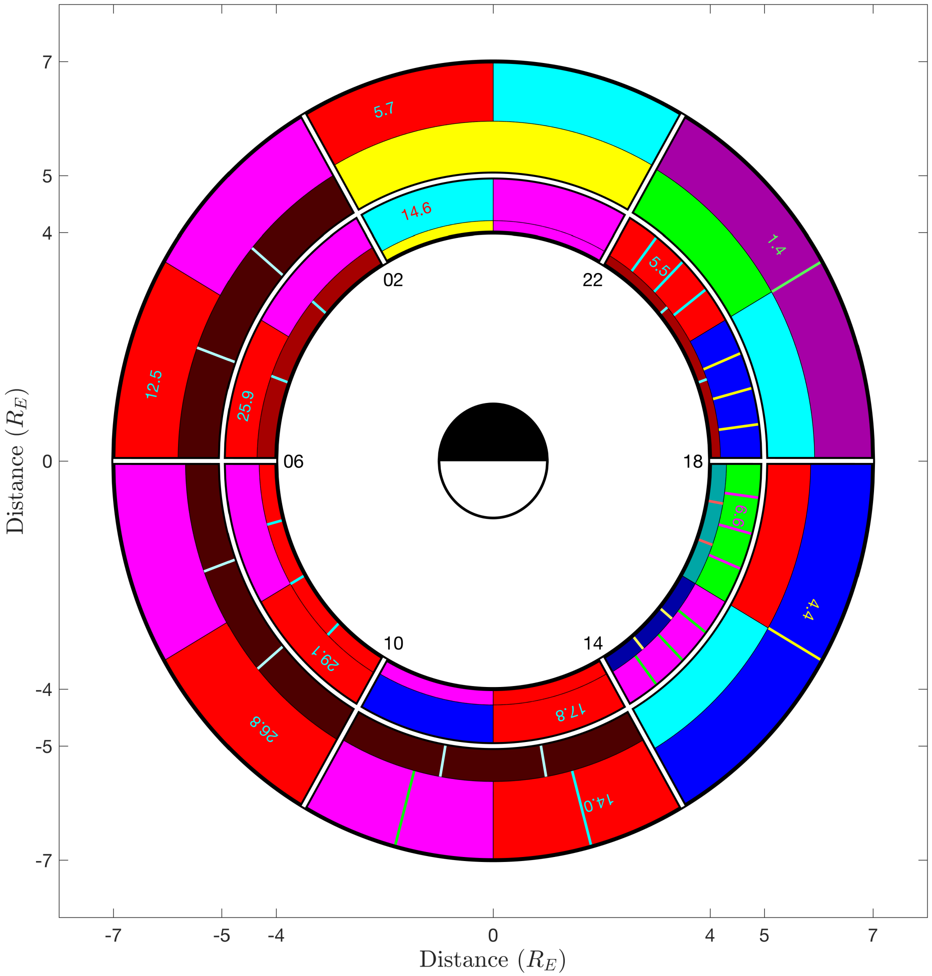
$$\begin{aligned} B_w(L, MLT, t) = F[& V(t-0), V(t-2), ..., V(t-20), \\ & n(t-0), n(t-2), ..., n(t-20), \\ & p(t-0), p(t-2), ..., p(t-20), \\ & B_B(t-0), B_B(t-2), ..., B_B(t-20), \\ & Dst(t-0), Dst(t-2), ..., Dst(t-20), \\ & AE(t-0), AE(t-2), ..., AE(t-20)] \end{aligned}$$

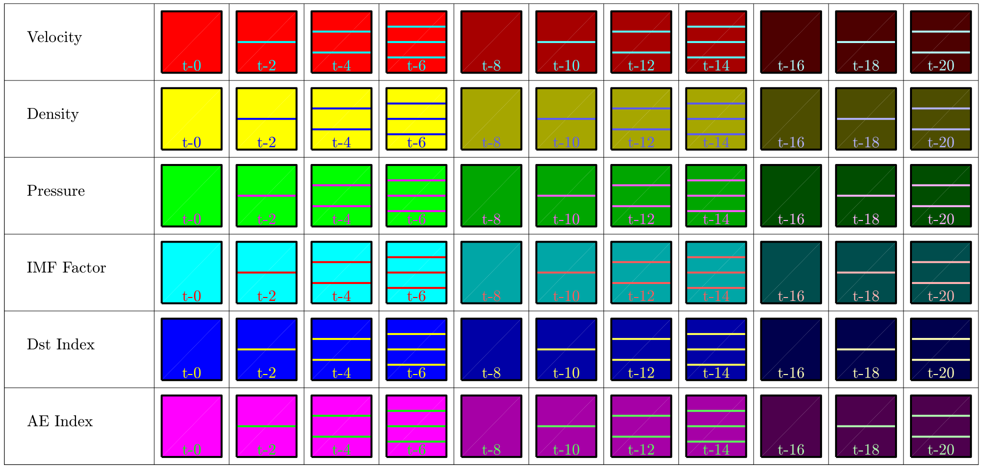
N.B. F contains no autoregressive or moving average terms as wave data is too sparse

ERR analysis of radiation belt waves: LBC waves

Quadratic *F*

LBC Waves: SW-C-GI inputs, v16 data: All Spacecraft

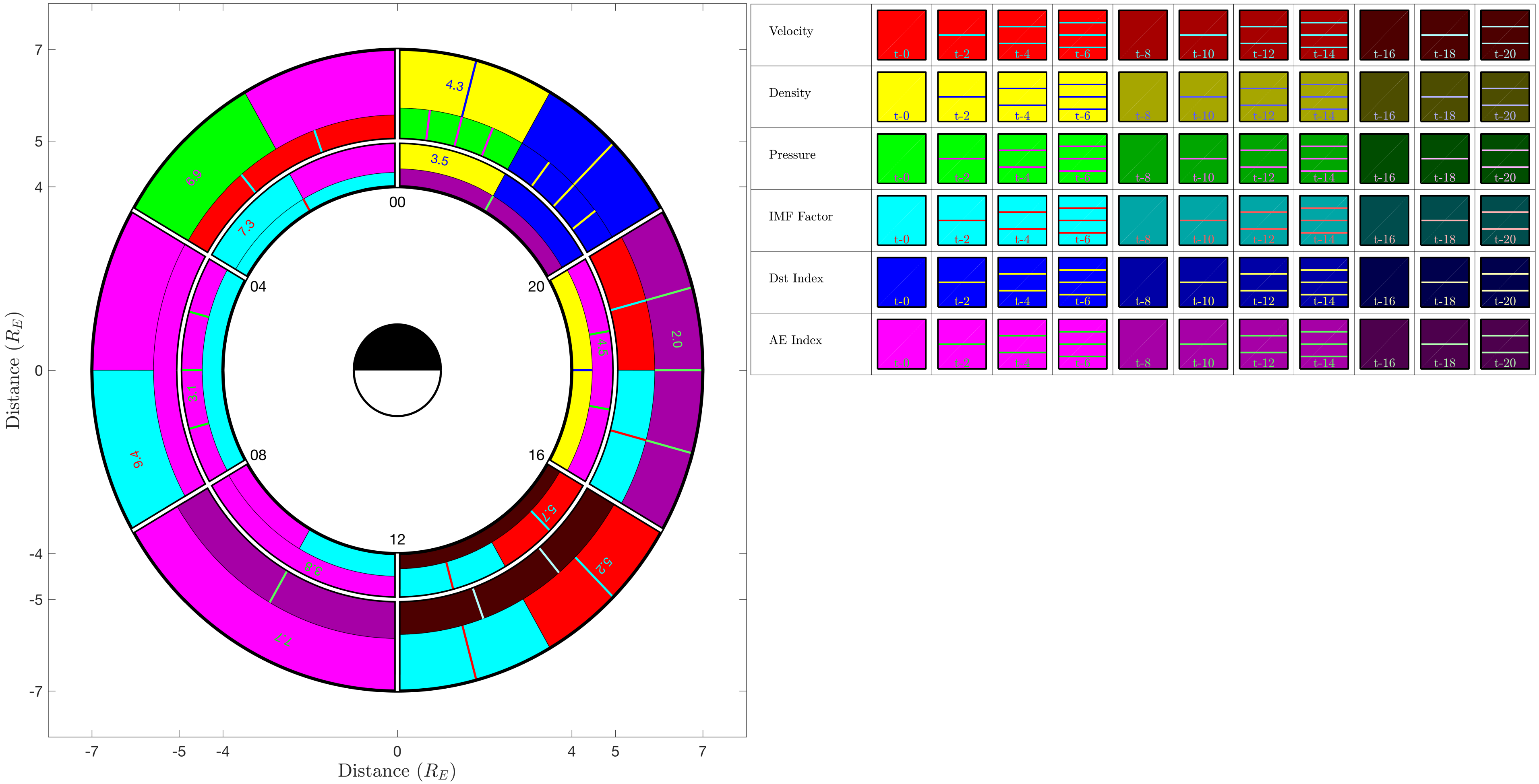




ERR analysis of radiation belt waves: Hiss waves

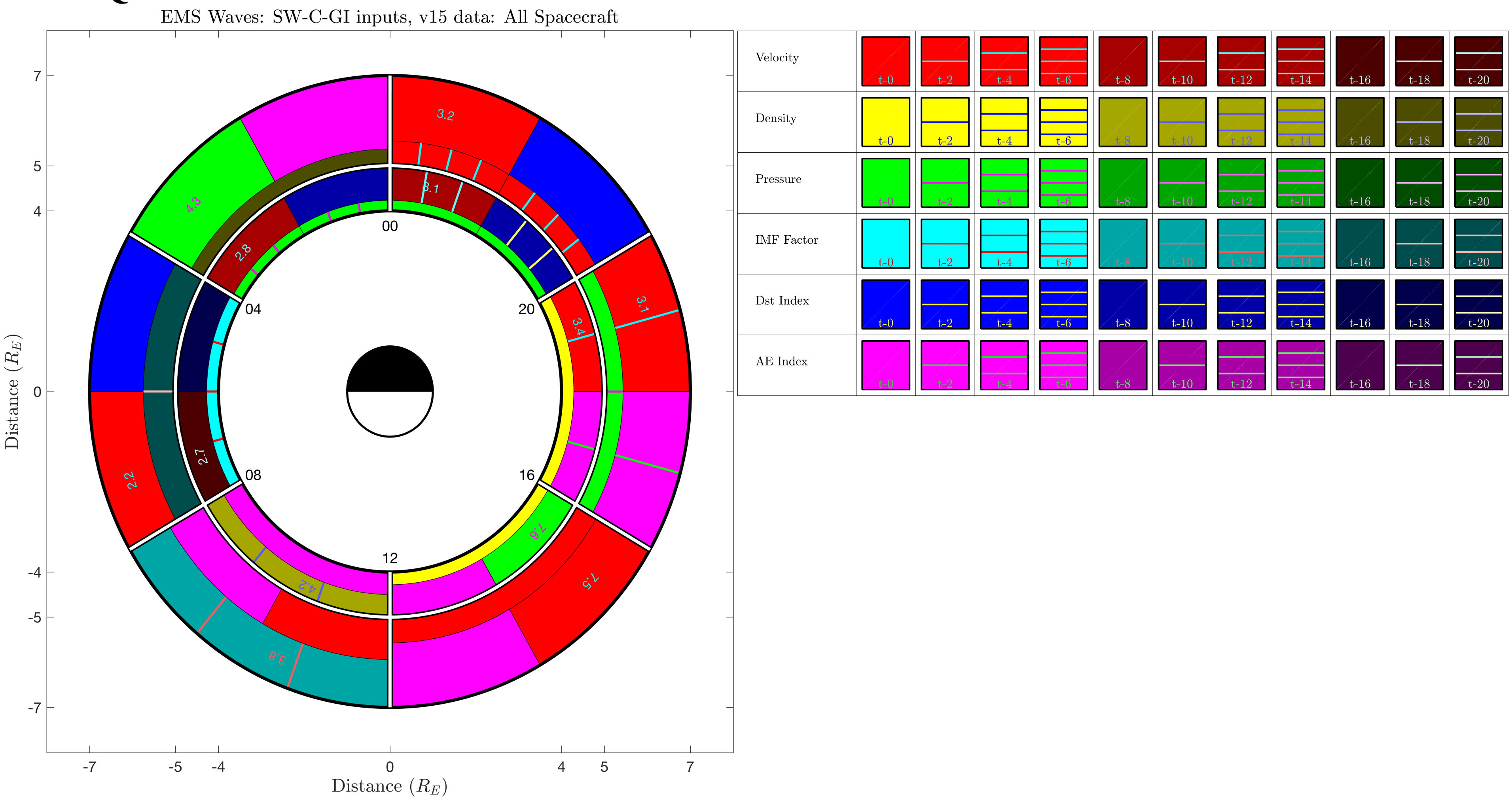
Quadratic *F*

Hiss Waves: SW-C-GI inputs, v15 data: All Spacecraft



ERR analysis of radiation belt waves: EMS waves

Quadratic *F*



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