

ROBUST NARMAX MODEL AND FORECAST OF GEOMAGNETIC INDICES

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Outline

- Introduction (Space Plasma Department, Department of Remote Sensing and Advanced Instrumentations)
- EU projects: EFFECTS (FP7), PROGRESS (HORIZON 2020)
- Developing a robust NARMAX model for operational use
- Dynamical-information approach to NARMAX and bilinear systems identification
- Estimation of Lyapunov exponents and Guaranteed prediction of geomagnetic indexes ₂

Introduction

1. This report starts with the physical basis and a brief description of the **system identification approach**. Following that, several examples illustrate practical issues in temporal and spatiotemporal prediction, **NARMAX** and **bilinear modeling**.

2. An approach based on **combination of nonlinear dynamical models and Lyapunov dimension** are used to analyze measurements of the geomagnetic indexes and solar wind parameters.

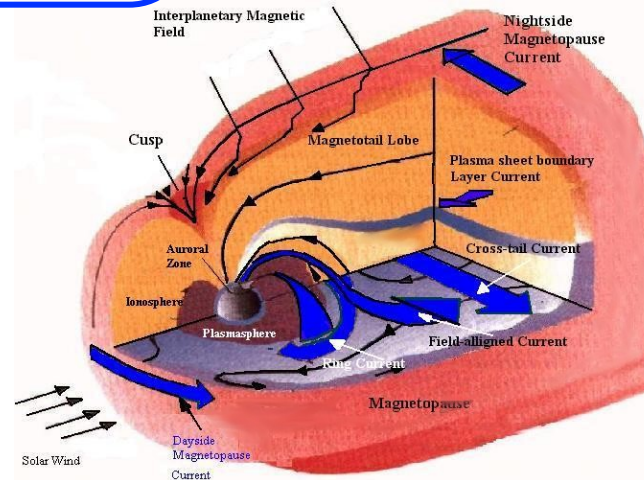
Dynamical-information forecasting of geomagnetic indexes

Magnetosphere is considered as a nonlinear complex dynamical system

Kp, AE, Dst indexes

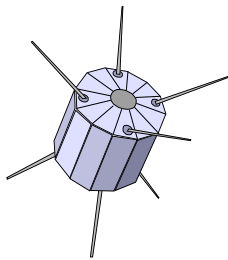


Solar wind parameters



Dst is sought for as an output of a nonlinear dynamical "black-box"

Data are from OMNI2 database:
<http://nssdc.gsfc.nasa.gov/omniweb/>
and Kyoto WDC for Geomagnetism:
<http://swdcdb.kugi.kyoto-u.ac.jp/>



Dynamical-information approach

- **Dynamical-information approach** is based on the "black-box" models and *Lyapunov exponents* to describe magnetospheric dynamics.
- **Reconstruction** of the dynamical model is based upon the application of **multiobjective learning algorithms** to identification of model's structure and parameters.
- A **forecasting algorithm** based on *Lyapunov exponents* is also proposed.

Objectives/Problems

1. Structure and parameter identification of NARMAX and bilinear models
2. Combination of NARMAX model and Lyapunov exponents
3. Combination of NARMAX model and Lyapunov dimension
4. Guaranteed prediction
5. Robust prediction.
6. Software implementation

Problem Description

Forecast of the evolution of geomagnetic indices

Our research concerns improvement and new development of models based on data driven modelling, such as BILINEAR and NARMAX (M. Balikchin S. Walker). Existing models for Dst and Kp will be analysed and verified with the aim of finding weaknesses and to suggest improvements. Solar wind and geomagnetic indices shall also be analysed in order to develop models for the identification of features, such as (but not limited to) shocks, sudden commencements, and substorms. Such categorisation will aid the model development and verification, and can also serve as alternative approach to models providing numerical input-output mapping. In addition to the development of Dst and Kp models new models will be developed to forecast AE.

Problem Description

- Recursive, robust bilinear dynamical model (RRBDM). It has minimal complexity and the same prediction limit as NARMAX. RRBDM provides forecasts of the Dst and Kp indices based on new robust algorithms and is driven by real time solar wind parameters measured at L1 with a time shift to account for the propagation of the solar wind to the terrestrial magnetopause and the real time Dst and Kp indices.

Problem Description

- The second, the **Guaranteed NARMAX Model** (GNM) also provides predictions of the Dst index. Its main advantage is that it delivers an increased **prediction reliability** in comparison to earlier SRI models.
- **Guaranteed prediction** of geomagnetic indexes

Problem (cont'd)

- A prediction methodology depends on the main Lyapunov exponent: a) if the main Lyapunov exponent positive we may use Lyapunov dimension for geomagnetic indexes prediction. b) if not, we may use NARMAX and the limit of predictability that based on Lyapunov exponents and their variations during coronal mass ejection

Papers and Conferences

- 1.V. Yatsenko, O. Cheremnykh, M. Balikhin, S. Walker. Prediction of geospace radiation environment and solar wind parameters: modeling, identification, and risk analysis.-Ukrainian Conference on Space Research.-Book Abstract, Odessa, Ukraine August, 22-27, 2016.-P.15.
- 2.V. Yatsenko. Space weather prediction using robust dynamical models: identification, optimization, and risk analysis.- 13th European Space Weather Week., November 14-17 2016, Oostende, Belgium,
- 3.V. Yatsenko. Space Weather Influence on Power Systems: Prediction, Risk Analysis, and Modeling. Geophysical Research Abstracts Vol. 18, EGU General Assembly, 2016.-P-6145.
- 4.V. Yatsenko. The influence of the free space environment on the superlight-weight thermal protection system: conception, methods, and risk analysis.- Automatic Welding, Paton Publishing House.-2016.-P.9 (in print).
- 5.V. Yatsenko, O. Cheremnykh. Kp index forecasting with robust dynamic models. - Journal "Automation and Information", Kiev, Ukraine, 2016.-12 P. (in print)

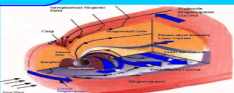
Vitaliy Yatsenko

The first, system science approach provides accurate forecasts of electron fluxes but is limited to regions in which continuous data are available, i.e. GEO. The second, based on physical principles, provides good coverage throughout the whole inner magnetosphere but with significantly lower accuracy. The third, based on new tools for modeling and system identification to prediction of risk using optimization methods. The combination of three approaches, as used in the SNB3GEO electron flux model (which combines the data driven NARMAX and physical VERB models), can overcome many of the shortcomings of the two individual models, generating improved short term forecasts for the whole RB region. Long term RB forecast require the estimation of solar wind parameters at L1 based on remote solar observations.

Dynamical-information forecasting of geomagnetic indexes

Magnetosphere is considered as a nonlinear complex dynamical system

Kp,AE,Dst indexes



Dst is sought for as an output of a nonlinear dynamical "black-box"



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<http://nssdc.gsfc.nasa.gov/omniweb/>
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Mathematical models

The **Guaranteed NARMAX Model (GNM)** provides predictions of the Dst index. Its main advantage is that it delivers an increased prediction reliability in comparison to earlier SRI models. **Guaranteed prediction of geomagnetic indexes**

Algorithms and software

- Algorithms and software for optimal structure and parameters identification of mathematical models of ionizing radiation have been considered.
- Forecasting mathematical models of ionizing radiation by numerical methods has been tested

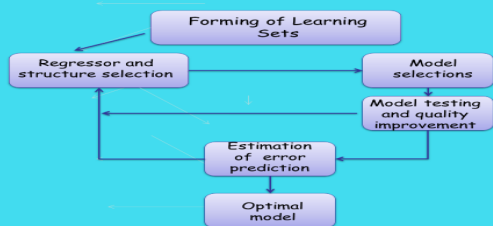


Fig. 1

Risk analysis

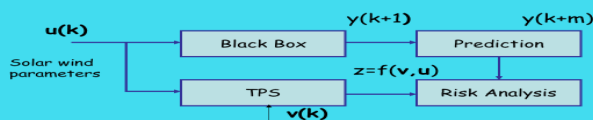


Fig. 2 Prediction and Risk Analysis

Optimization problem with constraints on risk

Let $z=f(v,u)$ be a loss function of a device depending upon the control vector v and a random vector u . The control vector v belongs to a feasible set V , satisfying imposed requirements. We assume that the random vector u has a probability density $p(u)$. We can define a function

$$\Phi_{\beta}(v, \beta) = (\alpha - \beta)^{-1} \int_{f(v,u) > \alpha} (f(v,u) - \alpha) p(u) du.$$

Optimization model

$$\min \mu(v)$$

$$v \in V, \Phi_{\beta}(x) \leq C_{\beta}, \Phi_{\gamma}(x) \leq C_{\gamma}.$$

Hybrid energy storage system based on supercapacitors

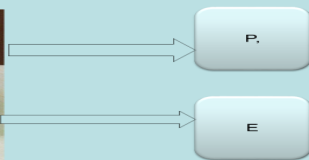


Fig. 3

Voltage decreases of supercapacitors before and after γ -irradiation

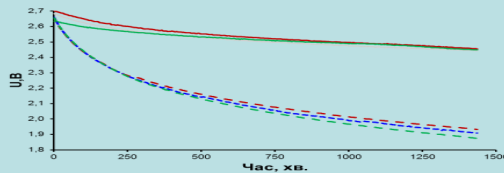


Fig. 4

Output of the diode laser after irradiation by gamma radiation

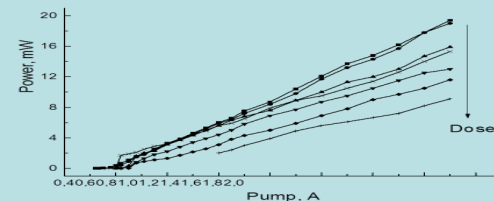


Fig. 5

Structure and parameter identification of NARMAX and bilinear models

$$y(t) = \sum_{i=1}^{\infty} \int_0^{\infty} \dots \int_0^{\infty} h_i(\tau_1, \dots, \tau_i) \prod_{j=1}^i u(t - \tau_j) d\tau_j$$

$$y(k) = F[y(k-1), \dots, y(k-n), u(k-1), \dots, u(k-n), \xi(k-1), \dots, \xi(k-n) + \xi(k)]$$

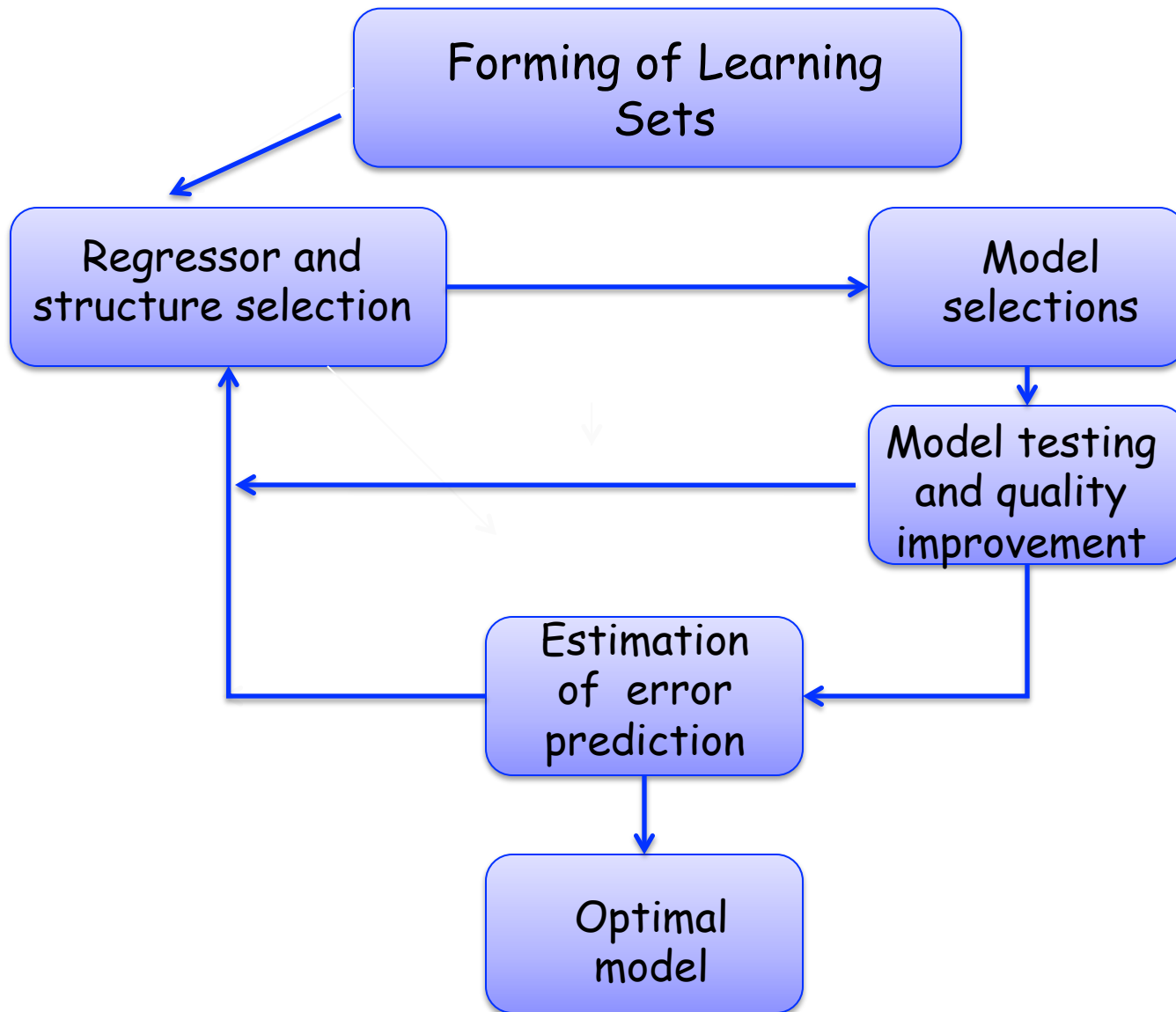
$$J = \sqrt{((\hat{y}(k) - y(k))^2) / (\sum (y(k) - \bar{y}(k))^2)}$$

Bilinear models

Methods of identifying bilinear systems from recorded input-output data have been proposed. The identified bilinear model is then used to forecast the evolution of the Dst index. For the investigation of **robust forecasting**, we perform a simulation study to demonstrate the applicability and the forecasting performance.

$$\dot{x}(t) = A_0 x + \sum_{i=1}^m B_i x(t) u_i(t),$$

$$y(t) = D x(t),$$



Optimization Problem

$$y(k) = \psi(k-1)^T \boldsymbol{\theta} + \xi(k) \quad (1)$$

$$\begin{aligned} &\min J(\boldsymbol{\theta}) \\ &\text{subject to } \boldsymbol{\theta} \in \mathcal{D} \end{aligned} \quad (2)$$

Optimization Problem

$$(P_M) \left\{ \begin{array}{l} \text{minimize } v(x) = \prod_{i=1}^m f_i(x) \\ \text{subject to } g_j(x) \leq 0, \quad j = 1, 2, \dots, p, \end{array} \right.$$

$$f_i : \mathbb{R}^n \rightarrow \mathbb{R} \quad (i = 1, 2, \dots, m) :$$

$$g_j : \mathbb{R}^n \rightarrow \mathbb{R} \quad (j = 1, 2, \dots, p)$$

$$\Omega := \{x \in \mathbb{R}^n : g_j(x) \leq 0, j = 1, 2, \dots, p\}$$

Numerical Algorithms

For structure and parameter identification we use the following numerical methods:

1. Nonlinear parametric model identification using genetic algorithms
2. Nonlinear optimization with constraints

Numerical results

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) + a_4 y(k-4) + a_5 y(k-5) + a_6 u(k-1) + \\ + a_7 u(k-2) + a_8 u(k-3) + a_9 u(k-4) + a_{10} u(k-5) + a_{11} u(k-6) + \\ + a_{12} u(k-4)u(k-5) + a_{13} y(k-1)u(k-4) + a_{14} u(k-5)u(k-3),$$

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) + a_4 y(k-4) + a_5 y(k-5) + \\ + a_6 u(k) + a_7 u(k-1) + a_8 u(k-2) + a_9 u(k-3) + a_{10} u(k-4) + \\ + a_{11} u(k-5) + a_{12} u(k-6) + a_{13} y(k-5) \cdot u(k-5) + a_{14} y(k-3) \cdot u(k-5) + \\ + a_{15} u^2(k-6),$$

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) + a_4 y(k-4) + a_5 y(k-5) + a_6 u(k-1) + \\ + a_7 u(k-2) + a_8 u(k-5) + a_9 y(k-2)u(k-1) + a_{10} u(k-4)u(k-6) + \\ + a_{11} y(k-3)u(k-1) + a_{12} u(k-1)u(k-7) + a_{13} y(k-3)u(k-2) + a_{14} u(k-2)u(k-5) + \\ + a_{15} u(k-7)u(k-12).$$

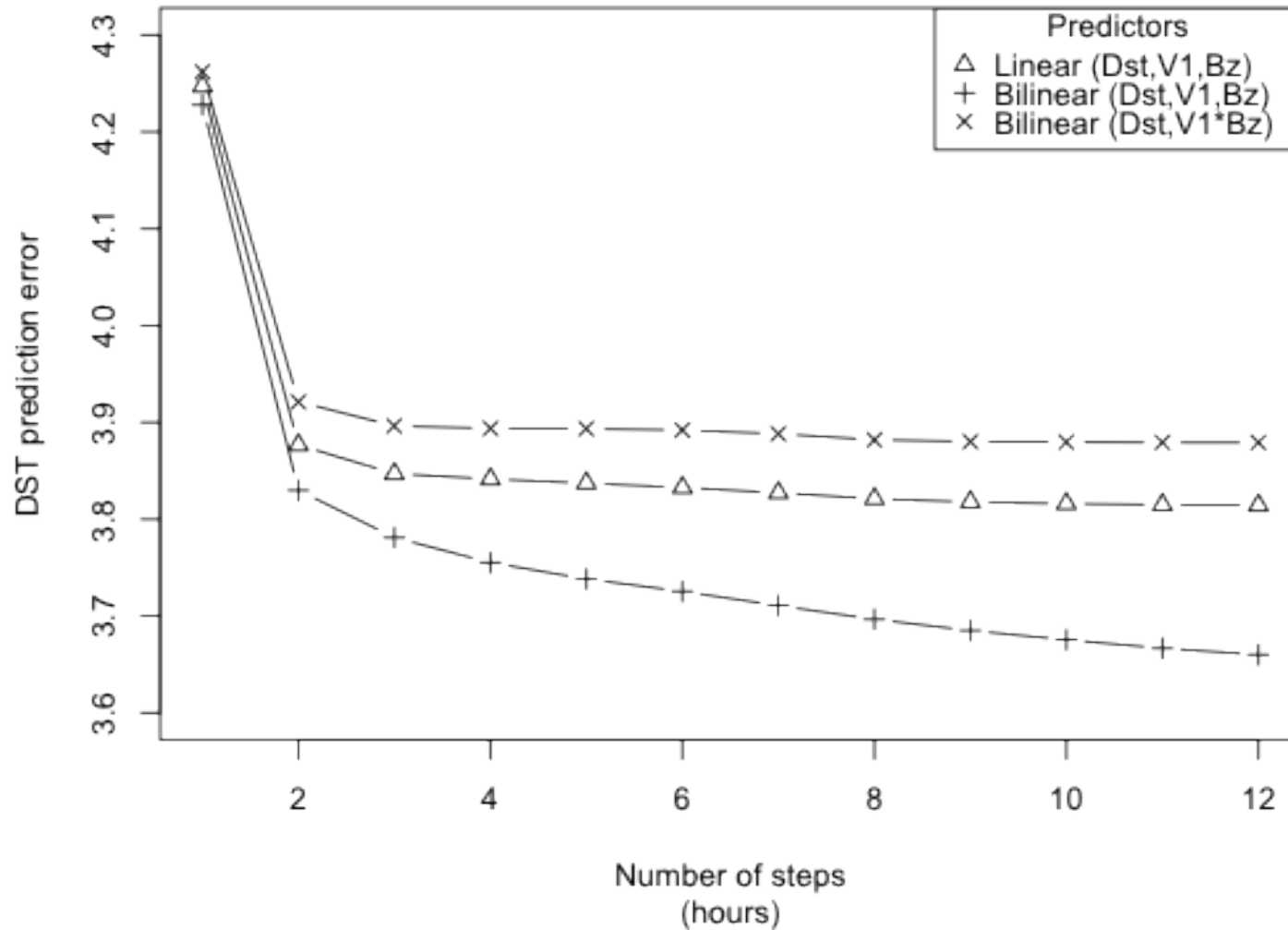
Numerical results

$$\begin{aligned}y_i = & 1.36 y_{i-1} - 4.99 u_{i-1} - 0.18 y_{i-2} u_{i-1} - 0.57 y_{i-4} - \\ & -1.43 u_{i-4} u_{i-6} - 0.75 y_{i-2} + 0.53 y_{i-3} + 0.1 y_{i-3} u_{i-1} + \\ & +0.36 y_{i-5} + 0.92 u_{i-6} u_{i-7} + 2.71 u_{i-2} + 0.08 y_{i-3} u_{i-2} + \\ & +0.78 u_{i-2} u_{i-5} - 0.91 u_{i-5} + 0.25 u_{i-7} u_{i-12}\end{aligned}$$

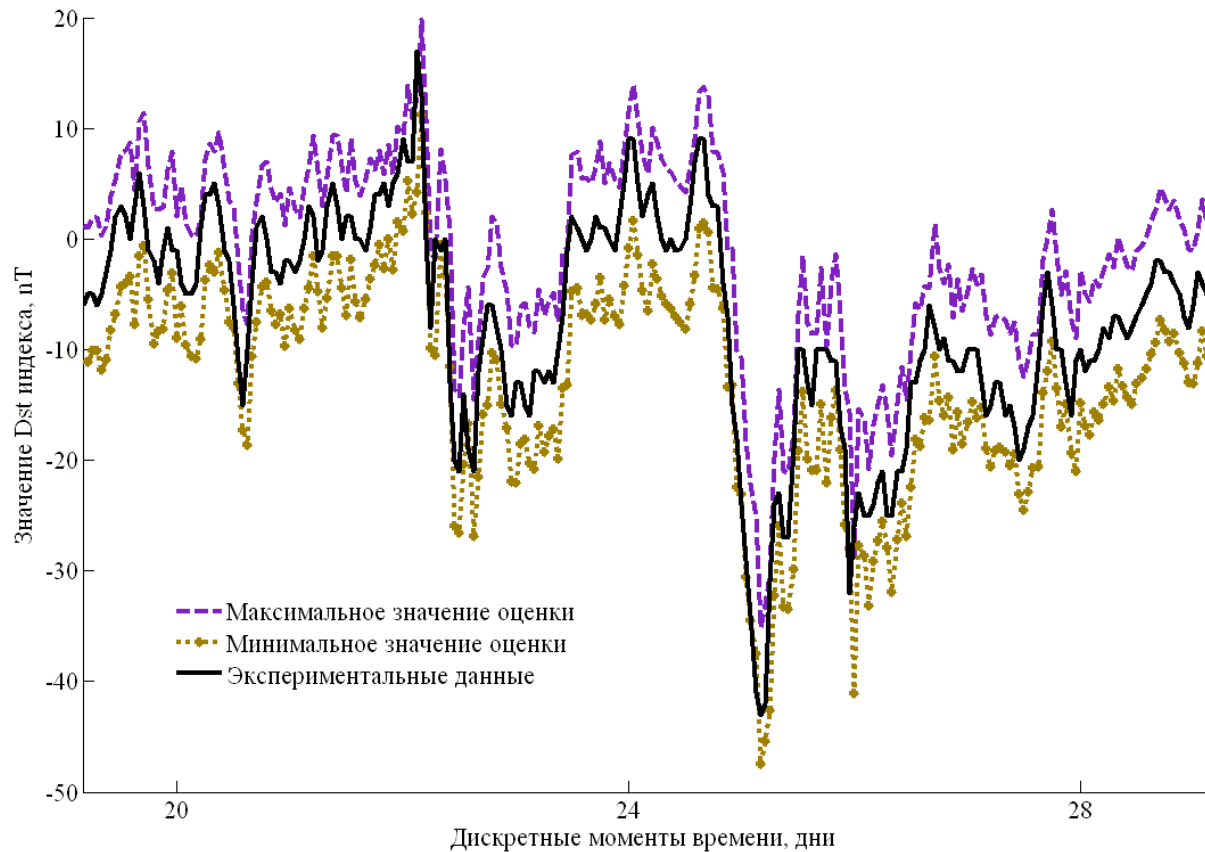
$$\mathbb{E} = \sqrt{\frac{((\hat{y}(k) - y(k))^2)}{(\sum (y(k) - \bar{y}(k))^2)}}$$

Numerical results

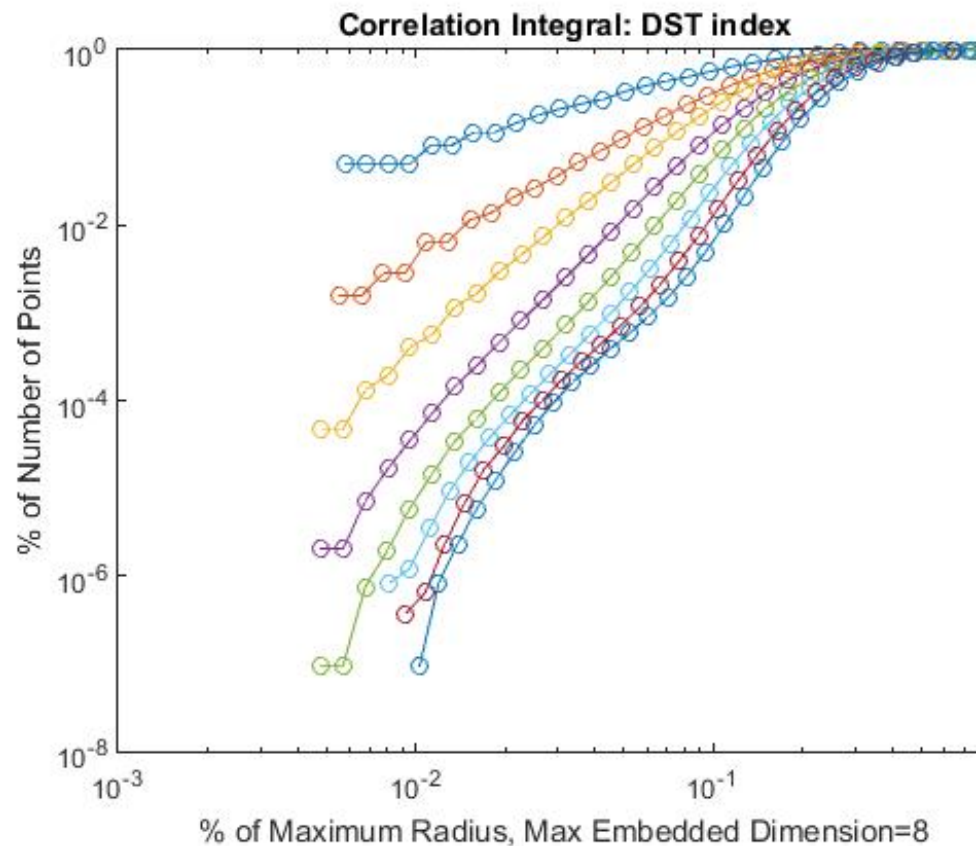
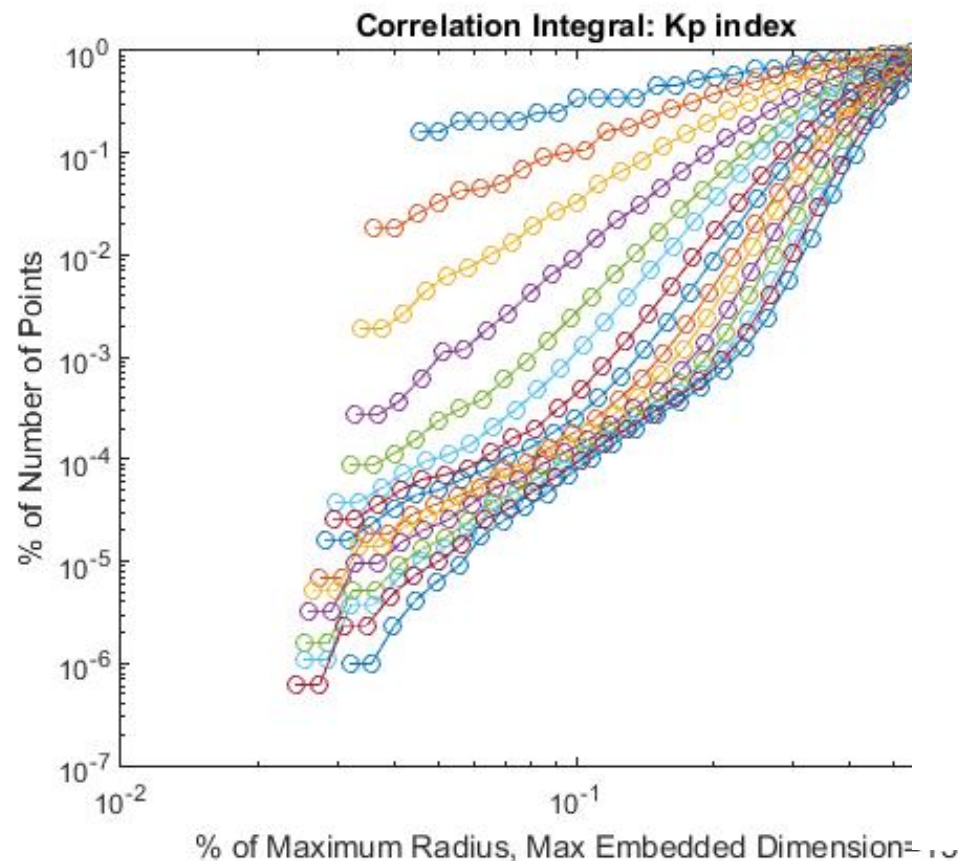
Mean square error for regressors



Results (Guaranteed prediction)

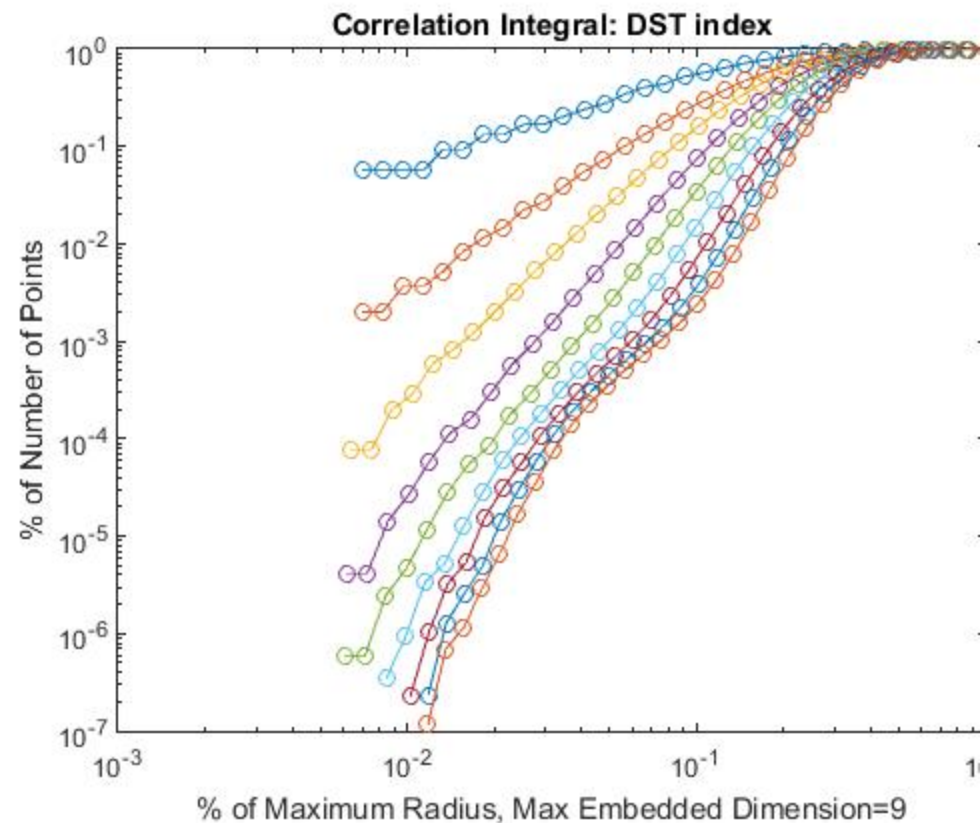
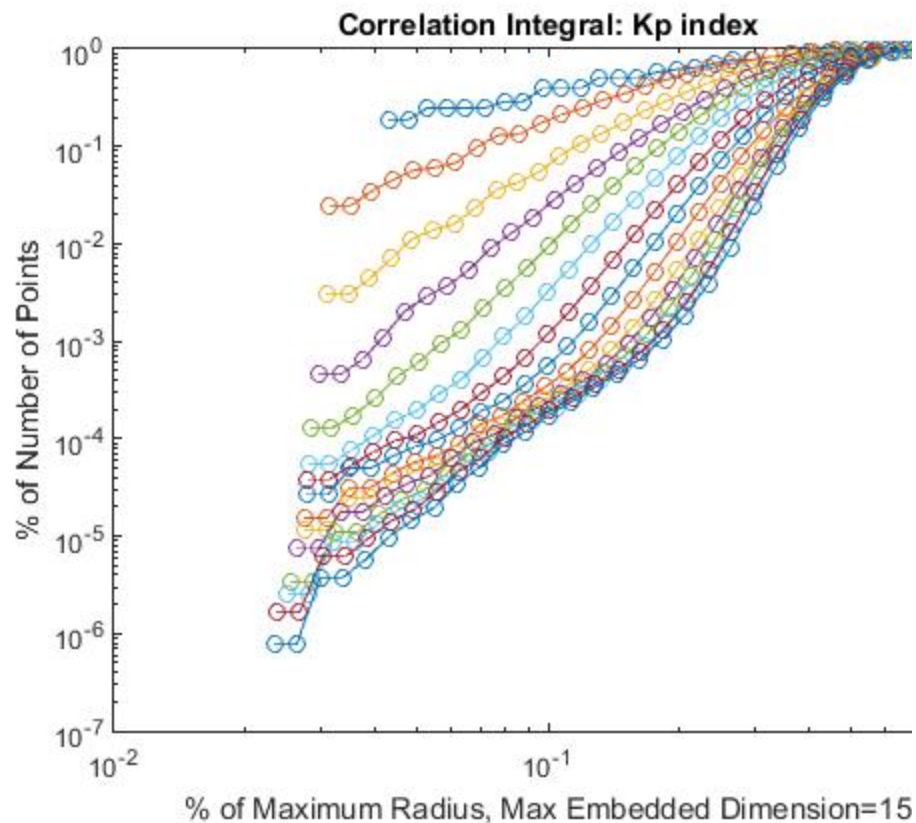


Correlation integrals of Kp & DST indices



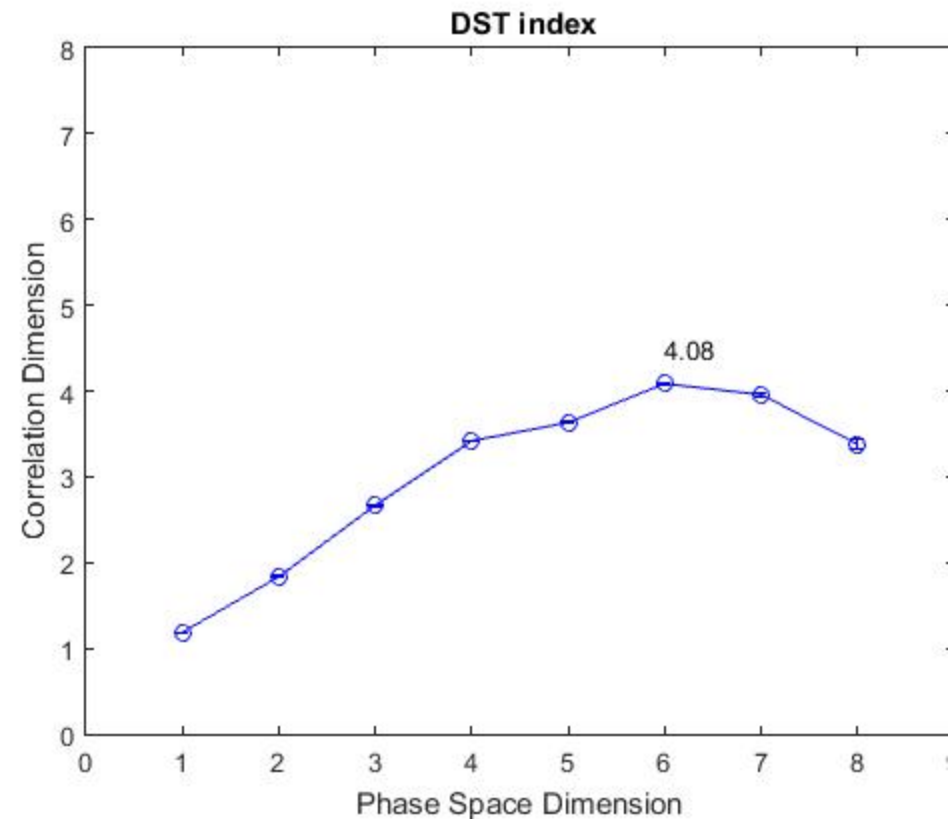
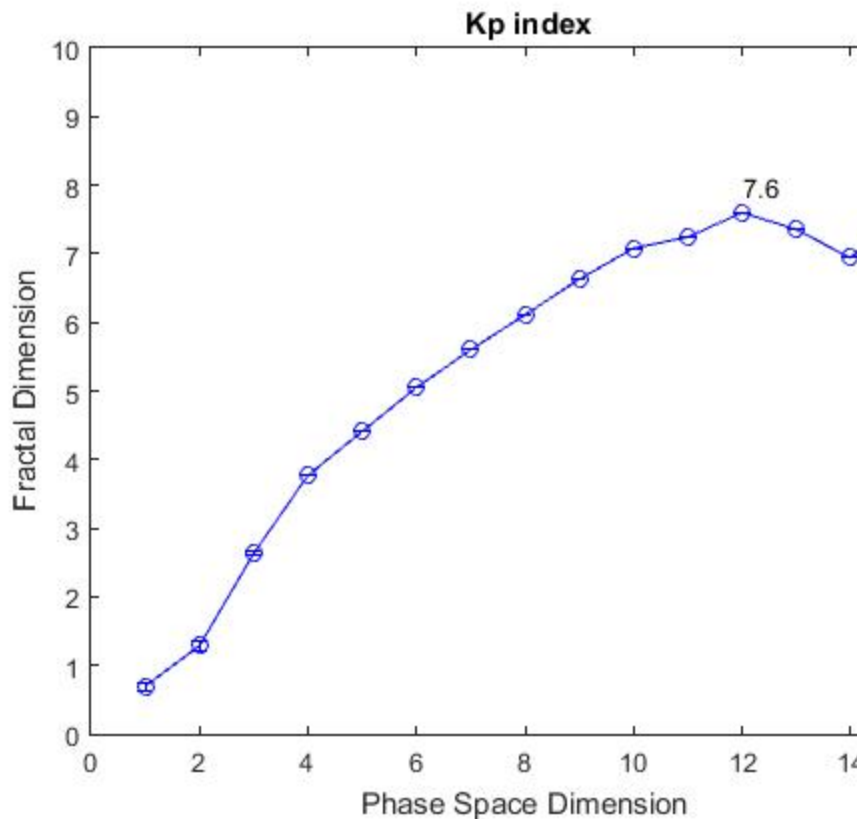
Data for: 06.2015 -04.2016.
 Kp(avg) ~ 20;
 DST(avg) ~ -18;

Correlation integrals of Kp & DST indices



Data for: 03.2013 -12.2013.
 Kp(avg) ~ 16;
 DST(avg) ~ -9,6;

Correlation and Phase dimensions of Kp & DST indices

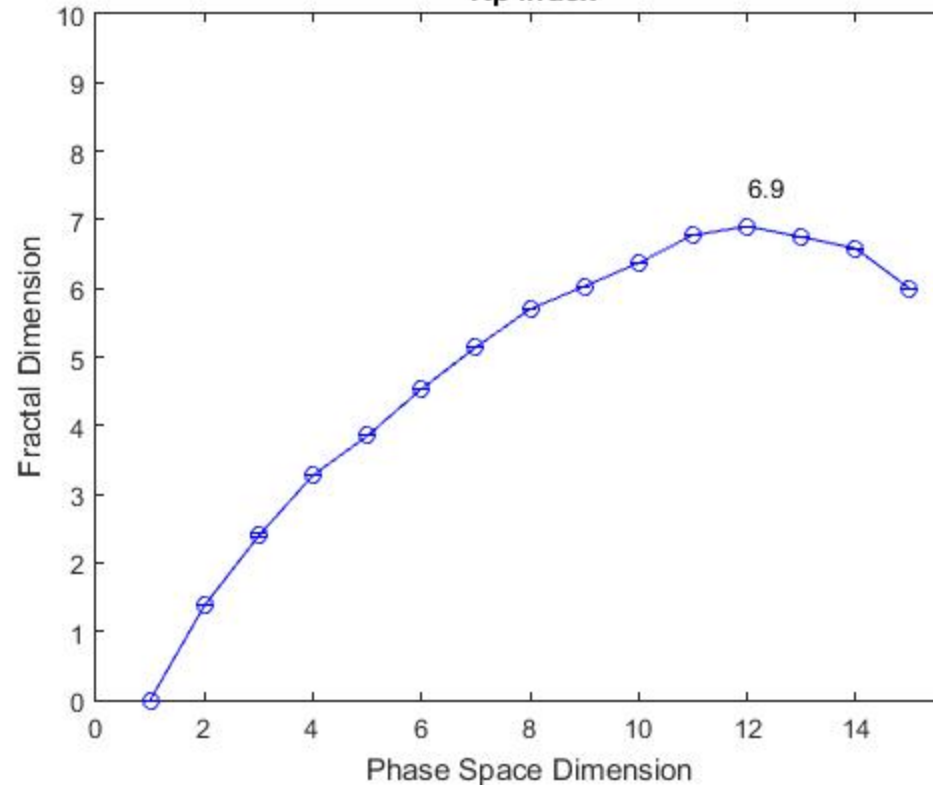


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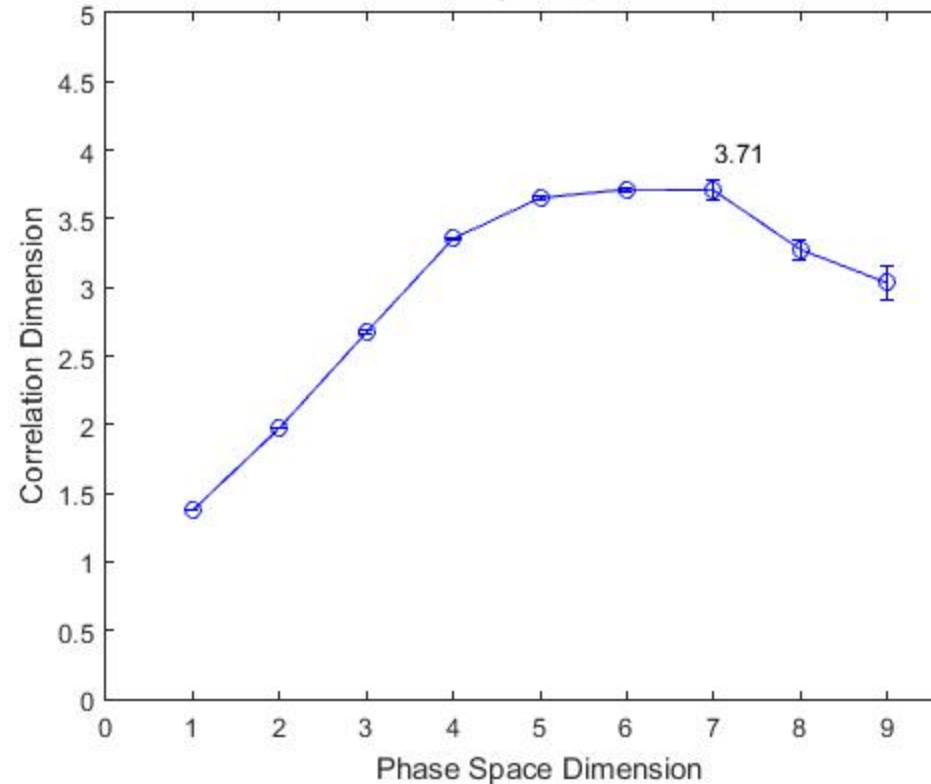
Correlation and Phase dimensions of Kp & DST indices



Kp index

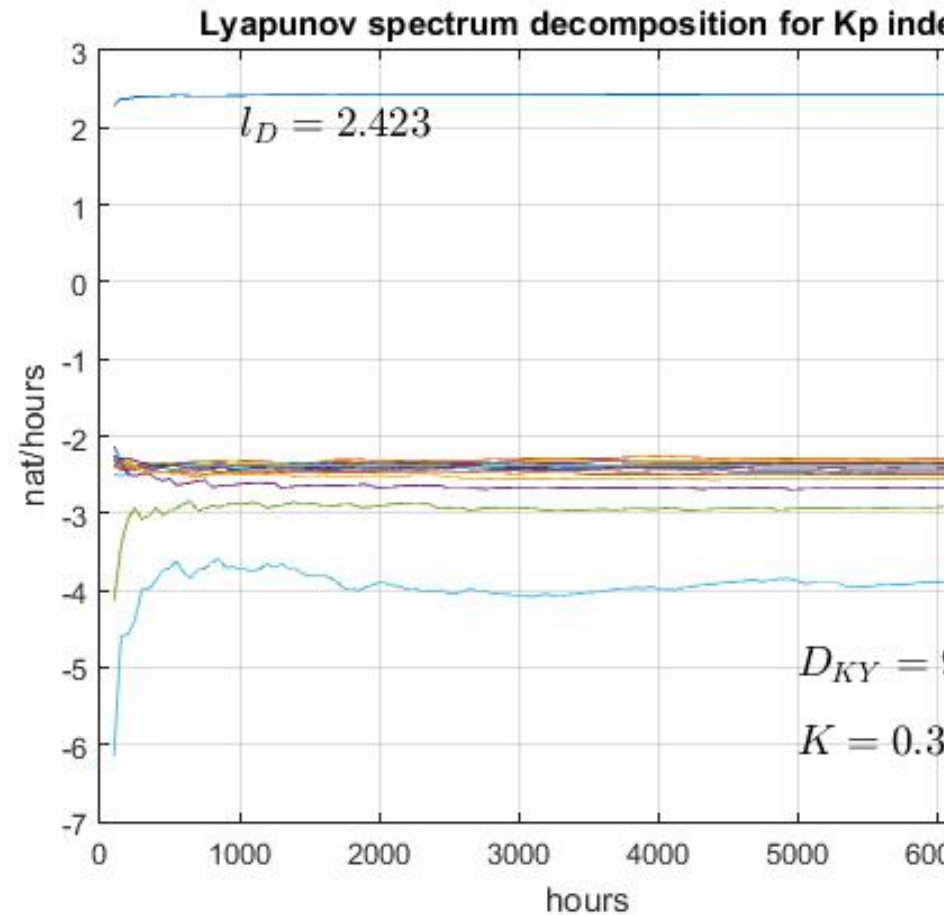
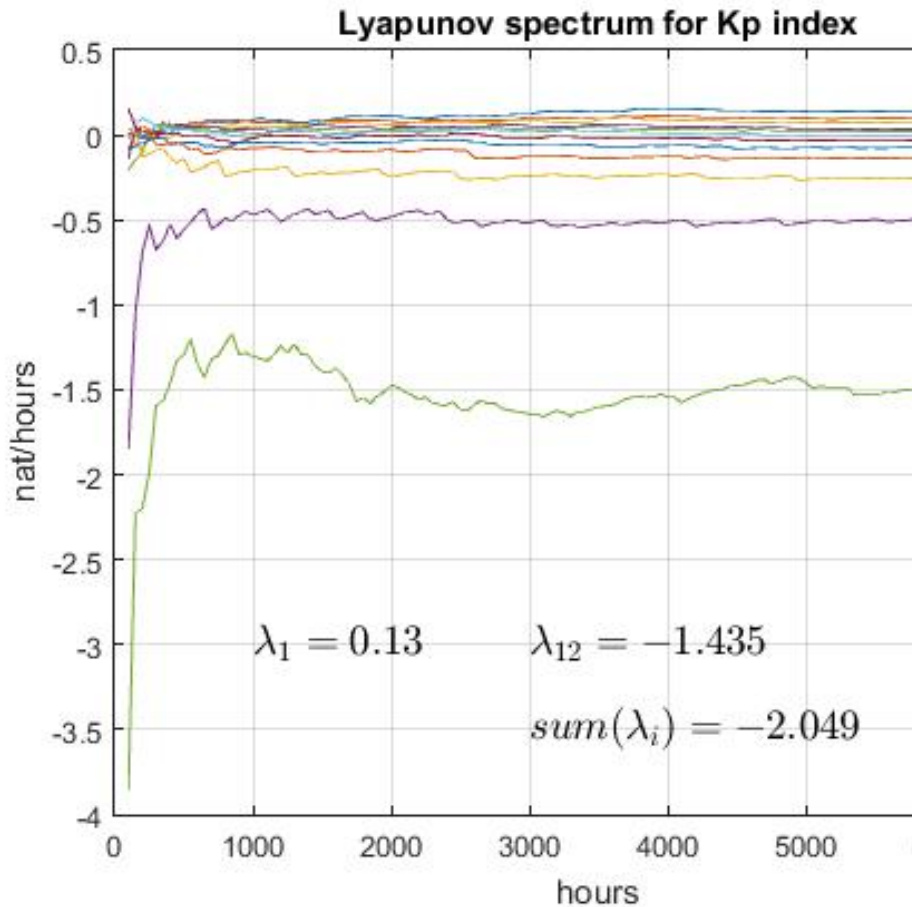


DST index



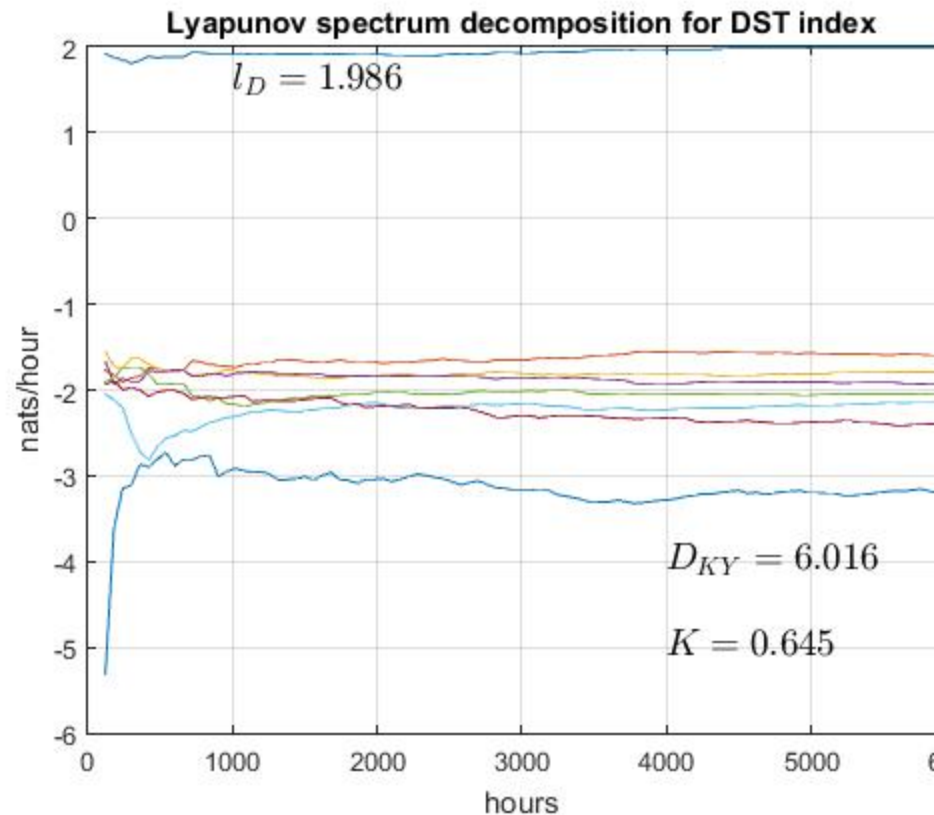
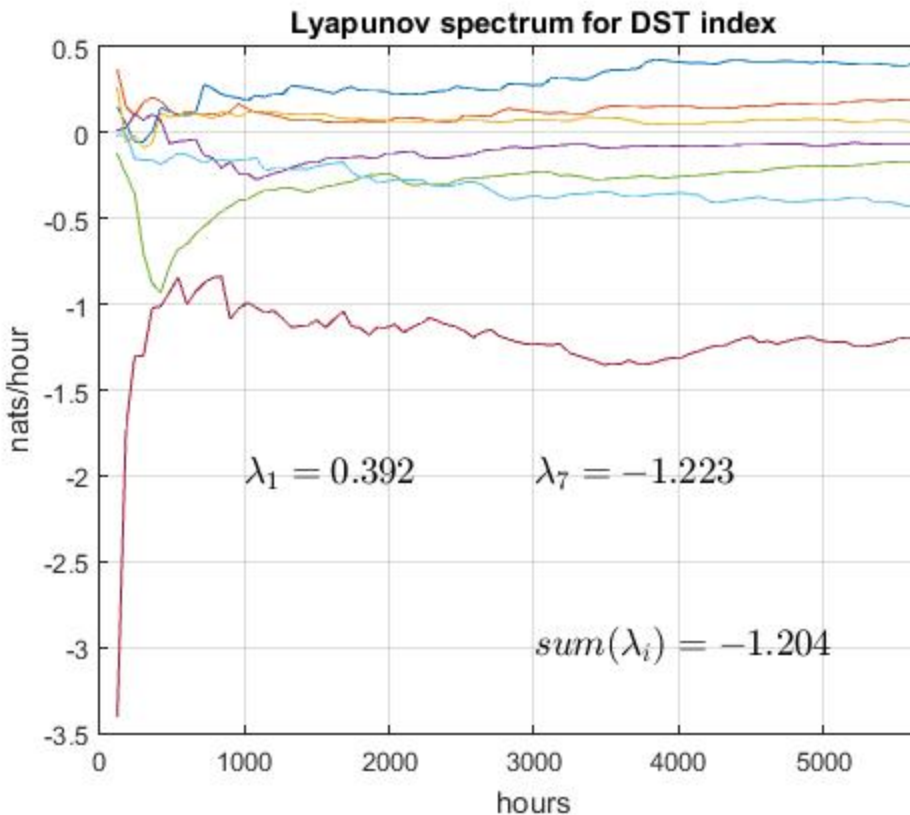
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Lyapunov spectrum decomposition



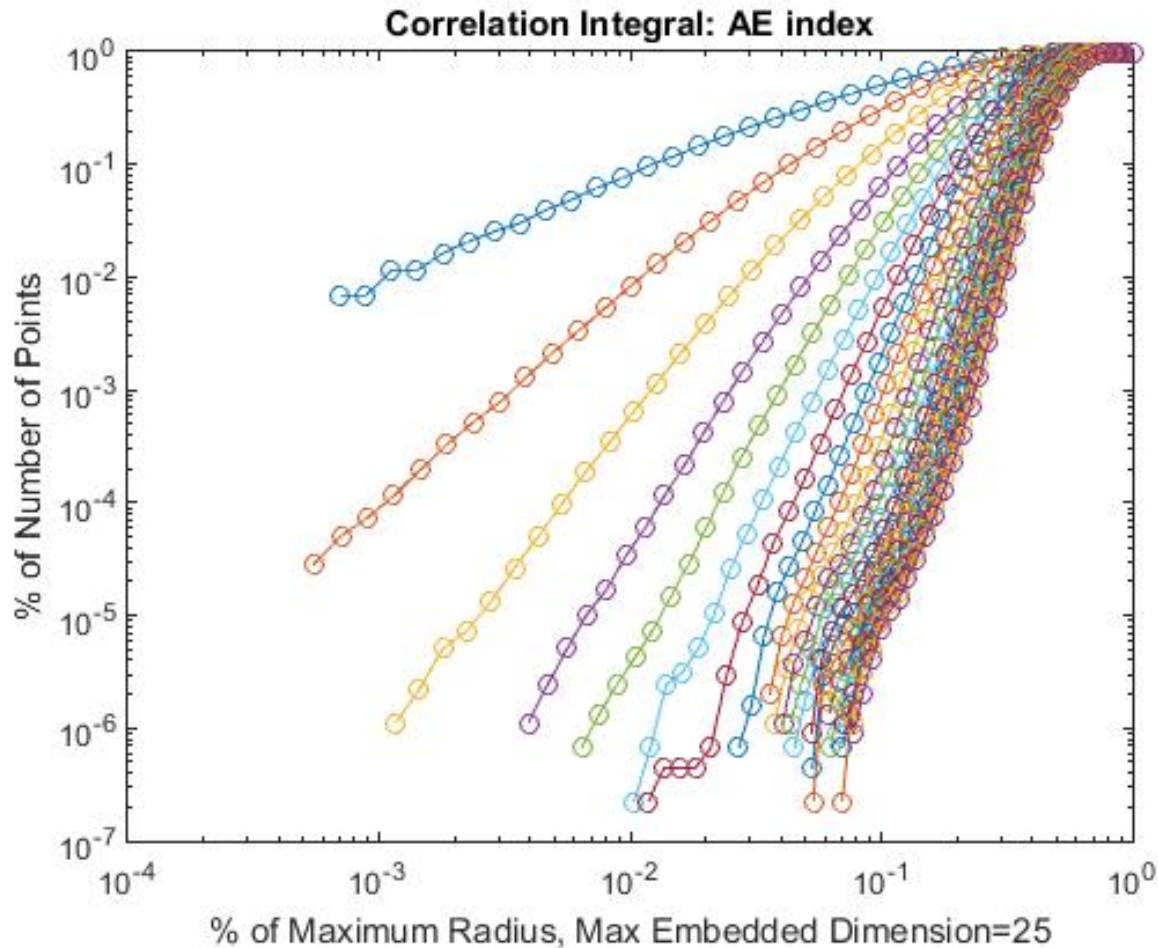
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Lyapunov spectrum decomposition



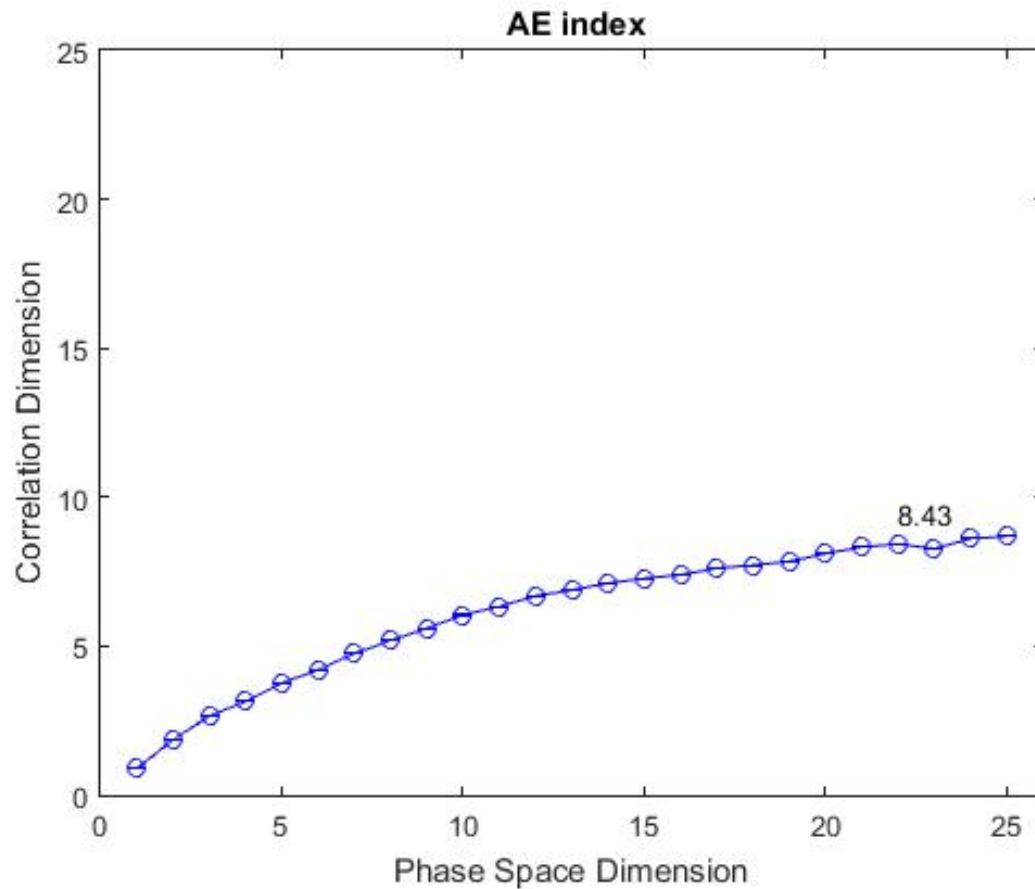
Data for: 06.2015 -04.2016.
Kp(avg) ~ 20;
DST(avg) ~ -18;

Correlation integral of AE index



Data for: 06.2015 -04.2016.
AE(avg) ~ 258 nT;

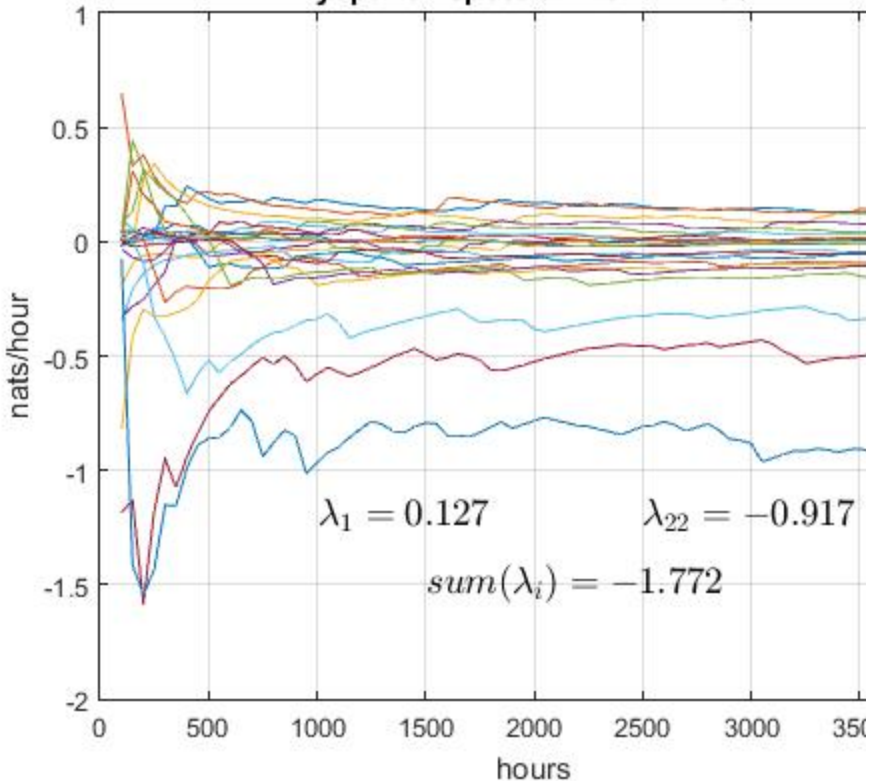
Correlation and Phase dimensions of AE index



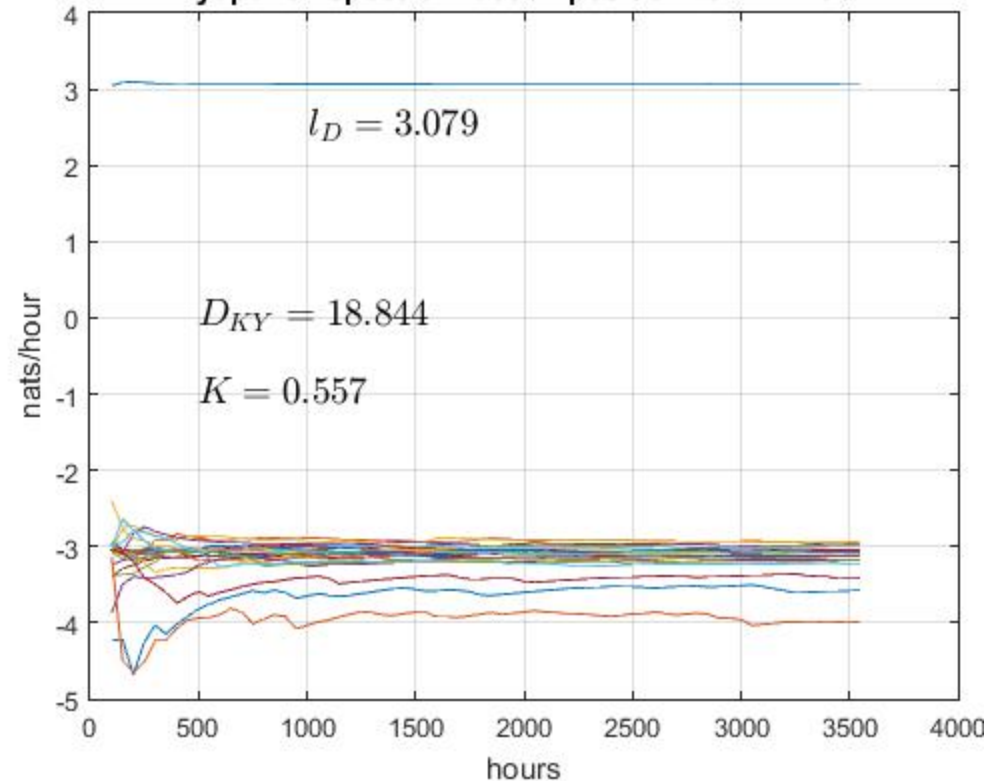
Data for: 06.2015 -04.2016.
AE(avg) ~ 258 nT;

Lyapunov spectrum decomposition for AE index

Lyapunov spectrum for AE index



Lyapunov spectrum decomposition for AE index



Data for: 06.2015 -04.2016.
AE(avg) ~ 258 nT;

Risk analysis

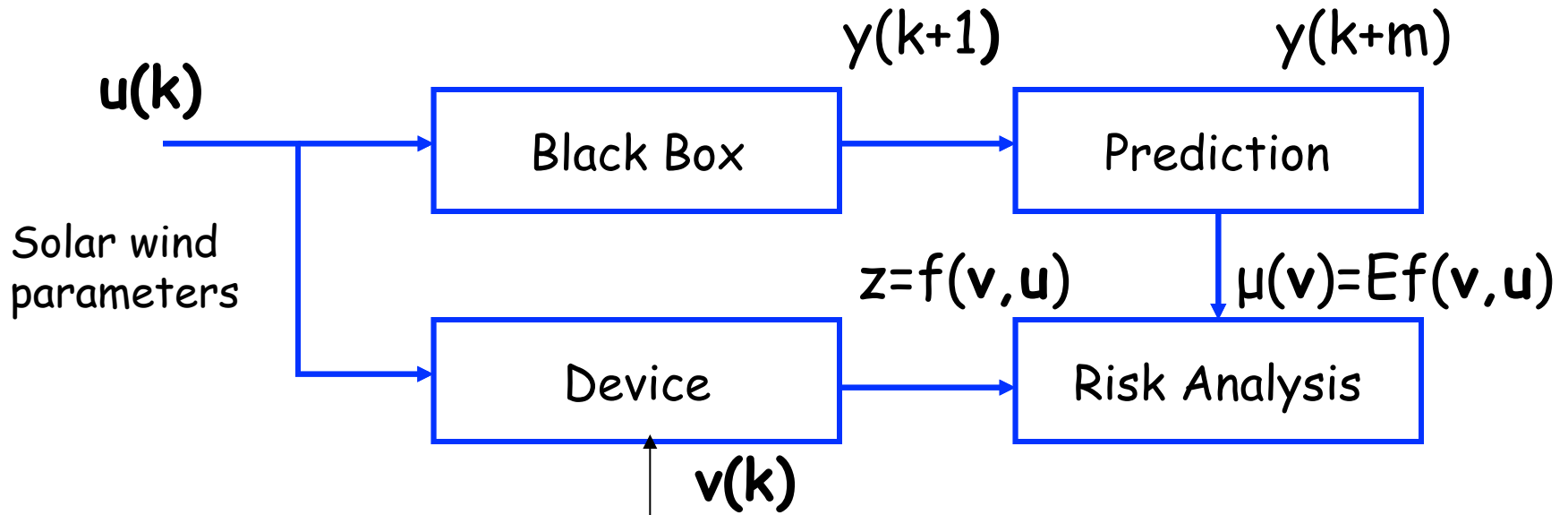


Fig. 1 Prediction and Risk Analysis

Optimization problem with constraints on risk

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$$\Phi_{\beta}(v, \beta) = (\alpha - \beta)^{-1} \int_{f(v,u) > \alpha} (f(v, u) - \alpha) p(u) du.$$

Optimization model

$$\min \mu(v)$$

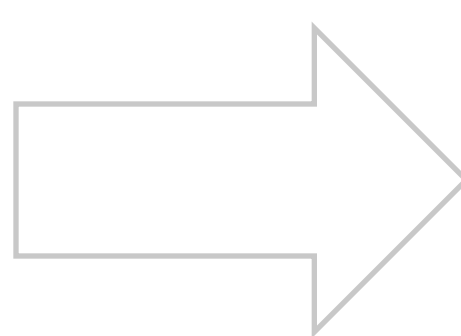
$$v \in V, \Phi_{\beta}(x) \leq C_{\beta}, \Phi_{\gamma}(x) \leq C_{\gamma}.$$

Applications

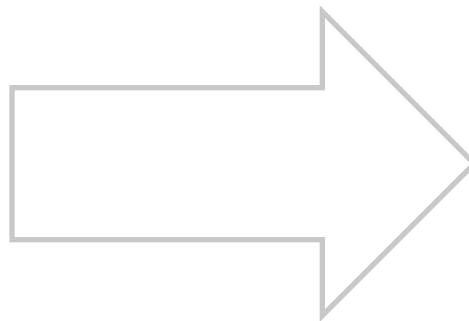
- Hybrid energy storage device based on supercapacitors
- Space accelerometers
- Superconducting gravimeter
- Lasers

Applications

Hybrid energy storage system based on supercapacitors

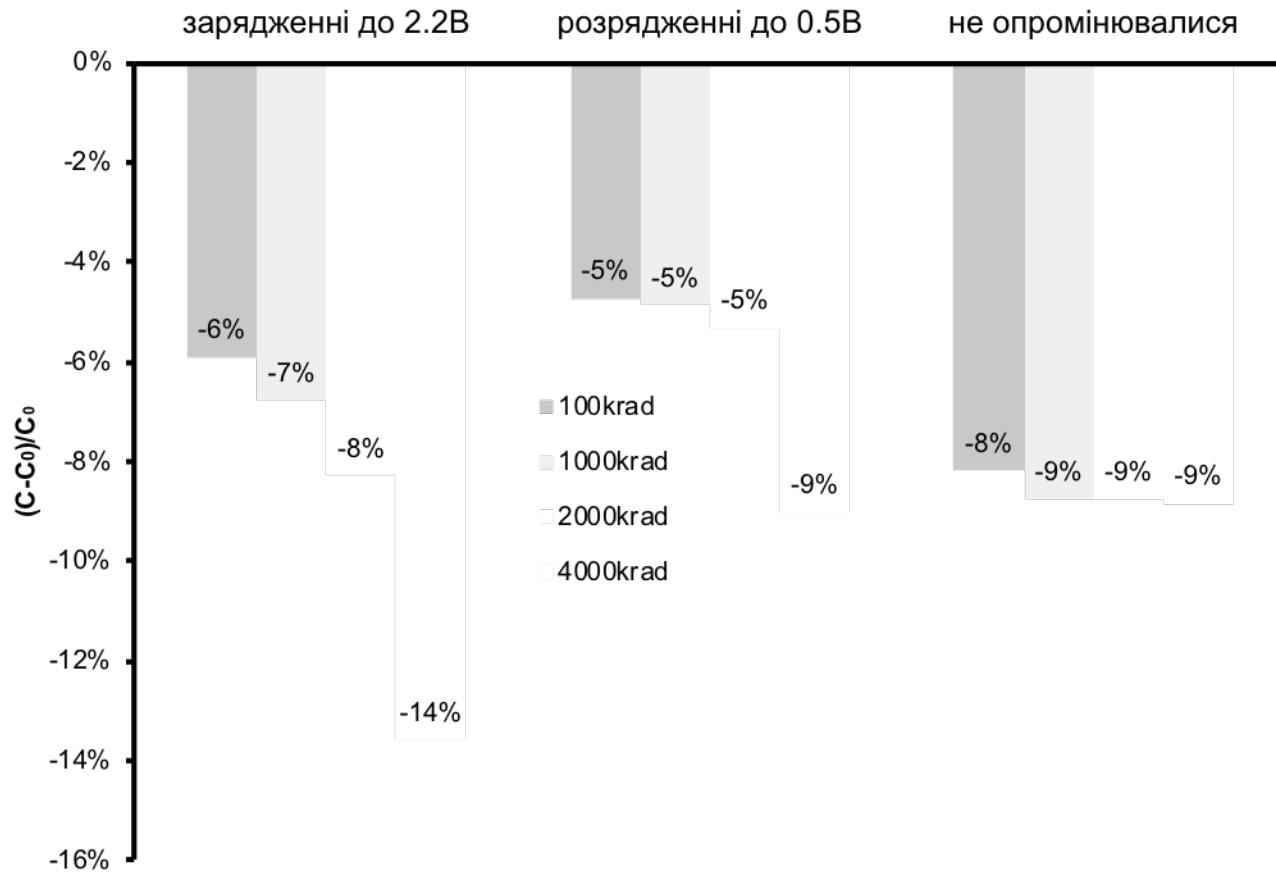


Р, кВт

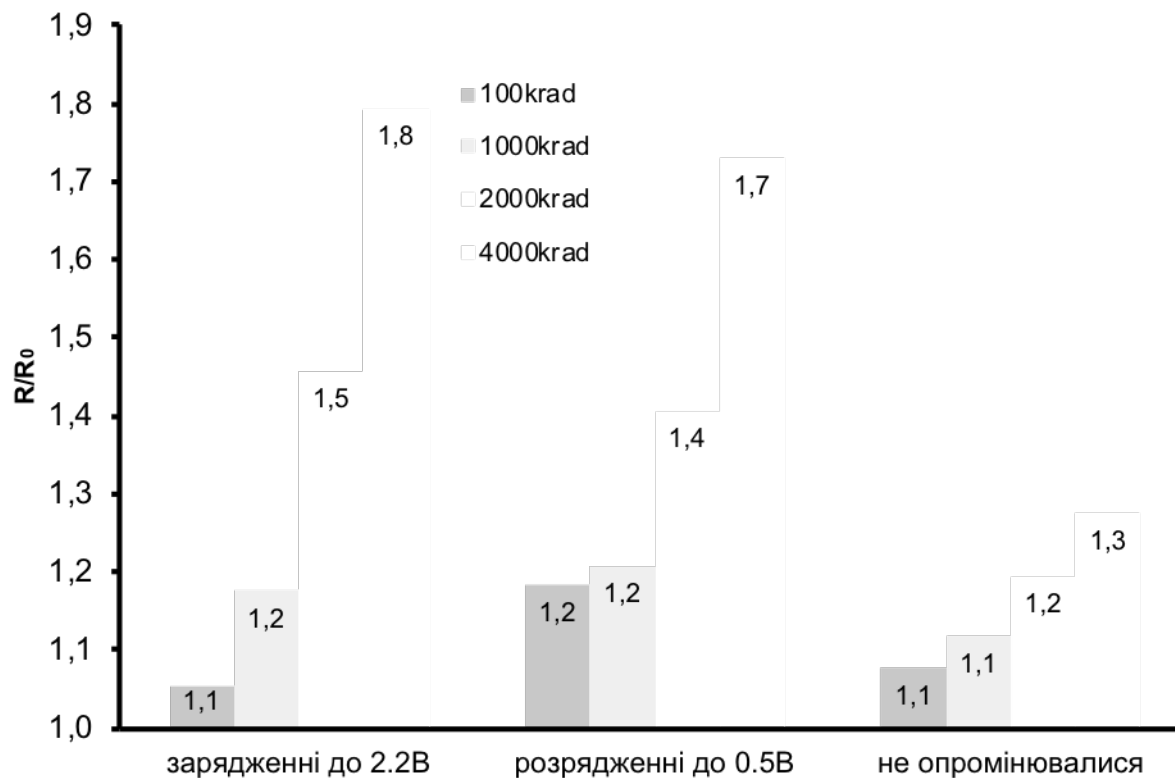


Е, Вт·год

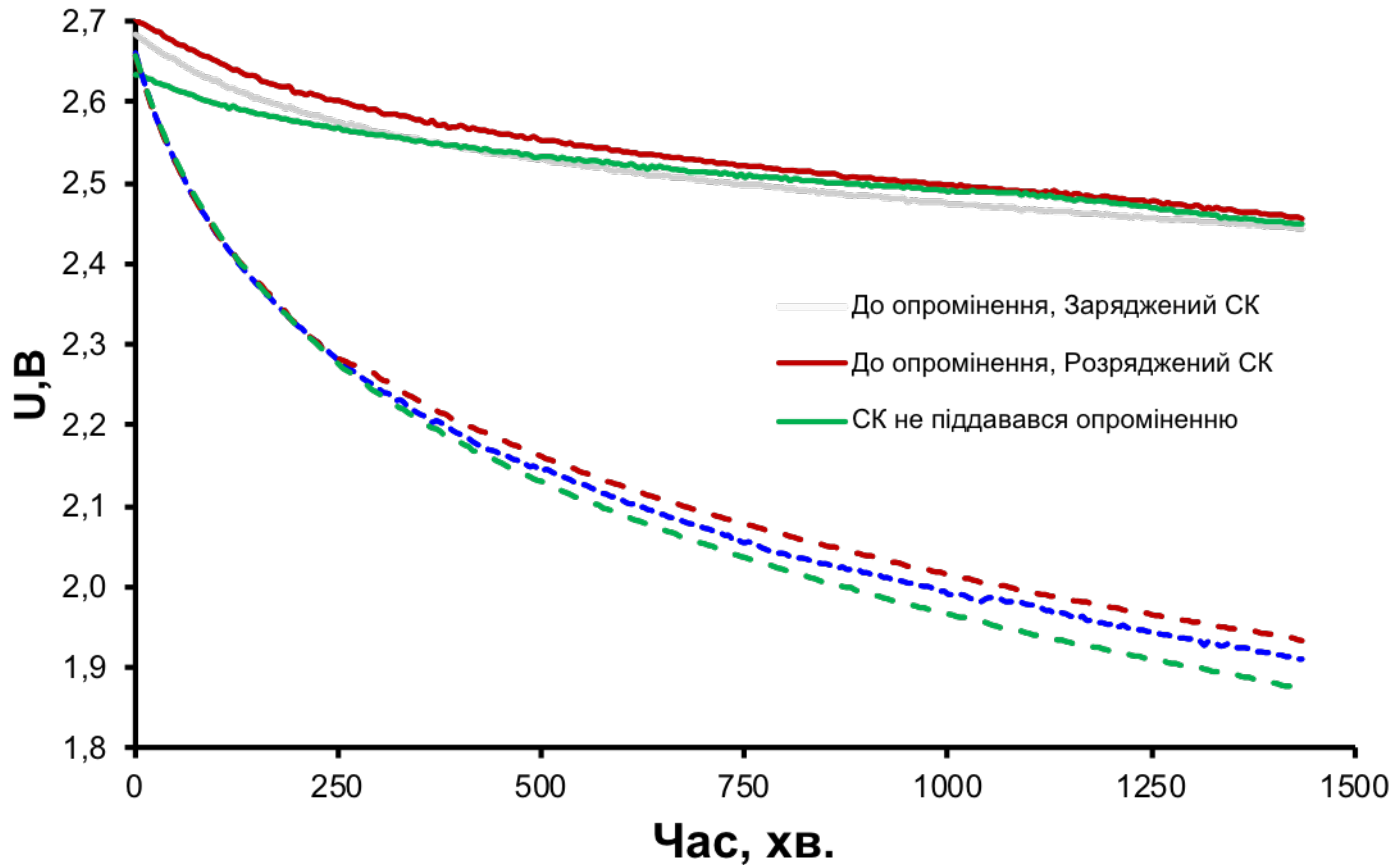
Impact γ -irradiation on capacity of hybrid energy storage device



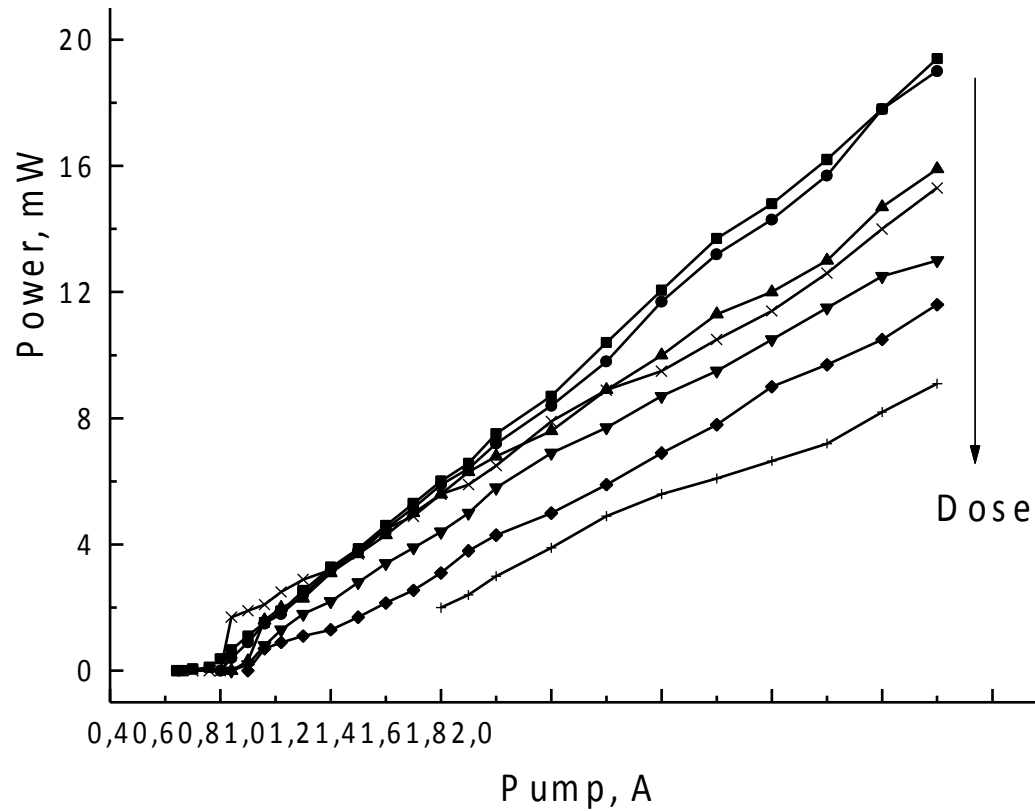
Resistance increase by γ -irradiation of hybrid energy storage device



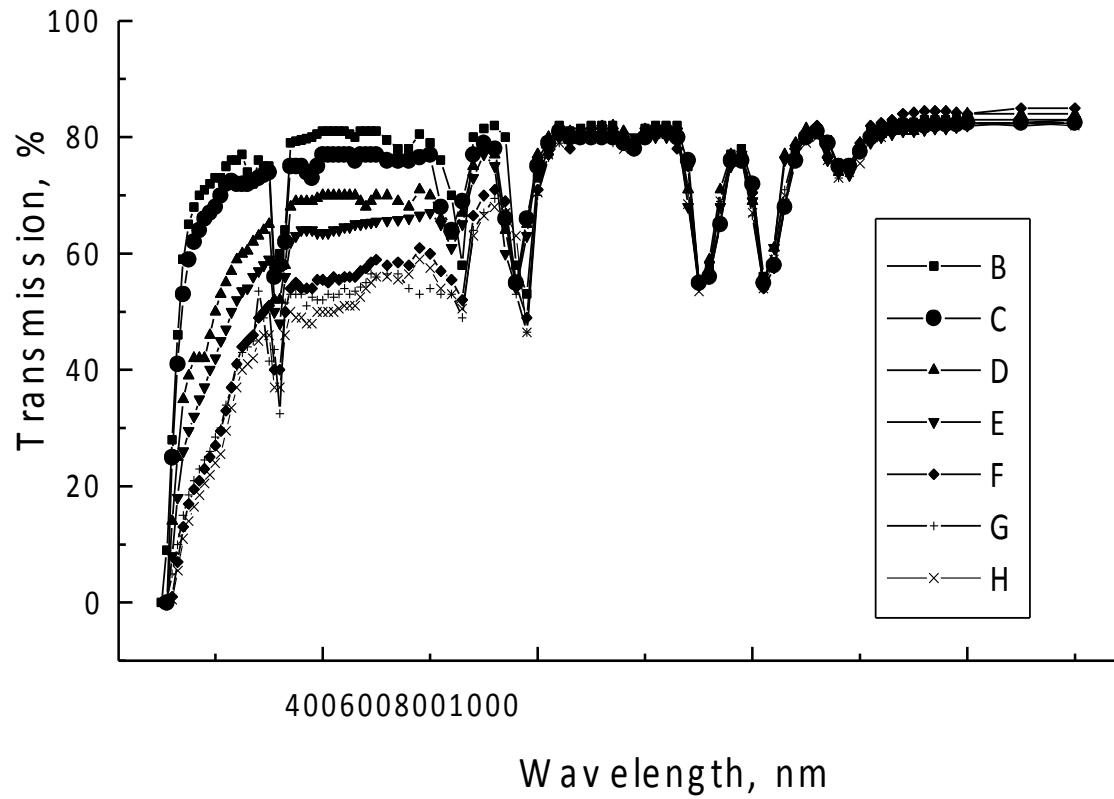
Voltage decreases of supercapacitors before and after γ -irradiation



Output of the diode laser after irradiation by gamma radiation



Transmission of Nd YAG crystal plate



Conclusions

- The following methods and models have been proposed:
 - (a) dynamical-information approach to NARMAX system identification;
 - (b) combination of NARMAX model and Lyapunov dimension;
 - (c) guaranteed prediction;
 - (d) robust models.
 - (e) risk assessment in safety analysis.

The End