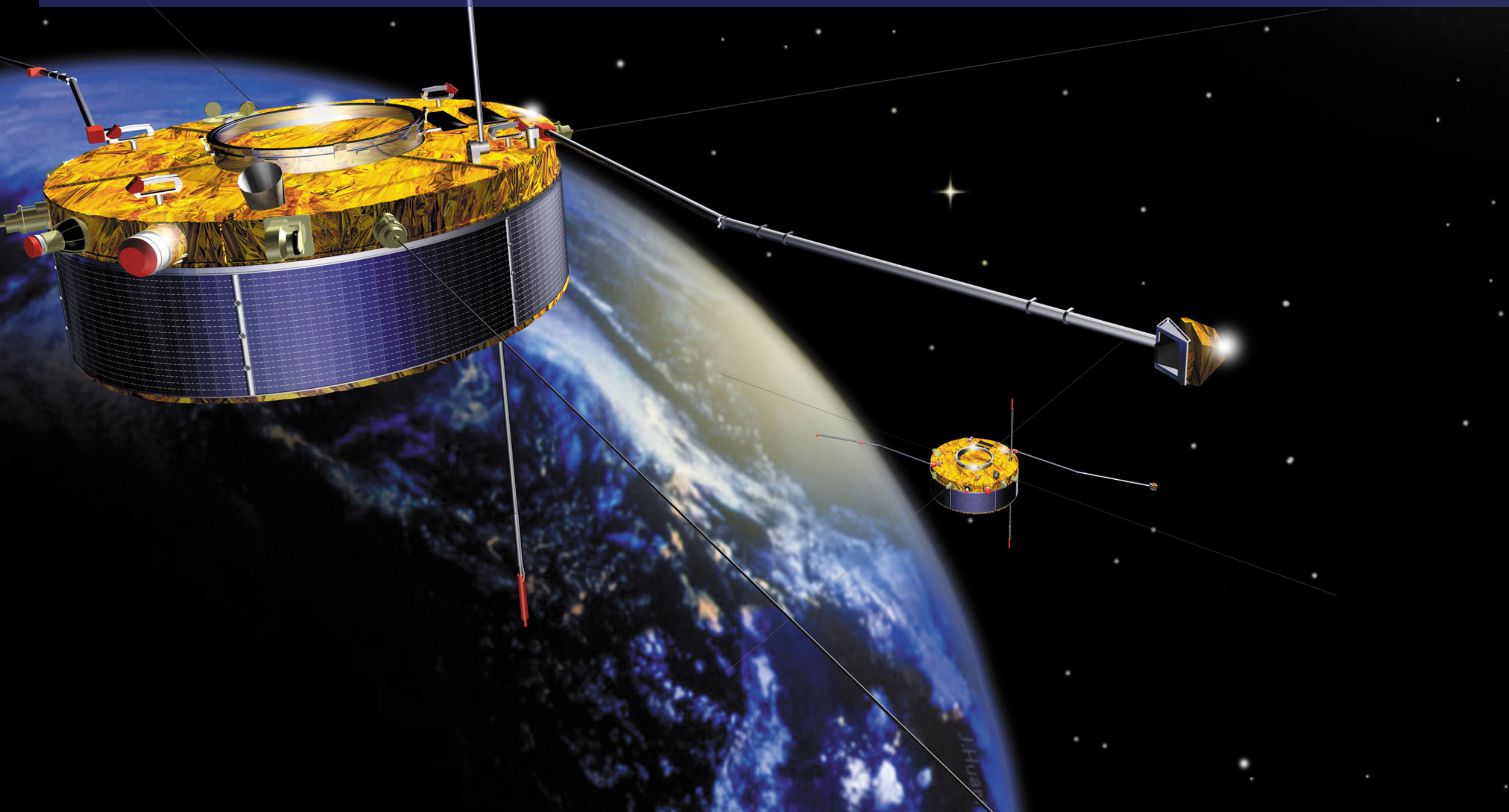
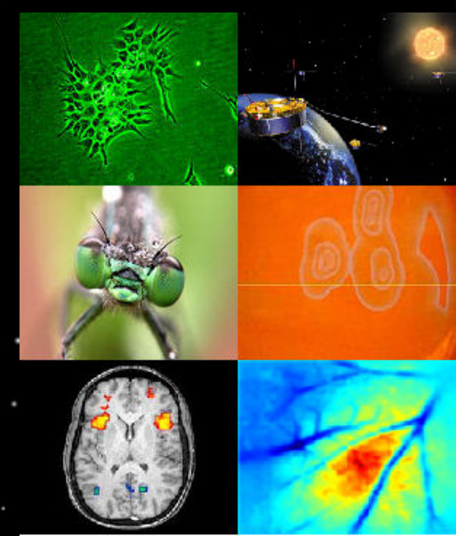
How the fusion between physics and systems science can help us to understand solar wind- magnetosphere coupling.





Participants

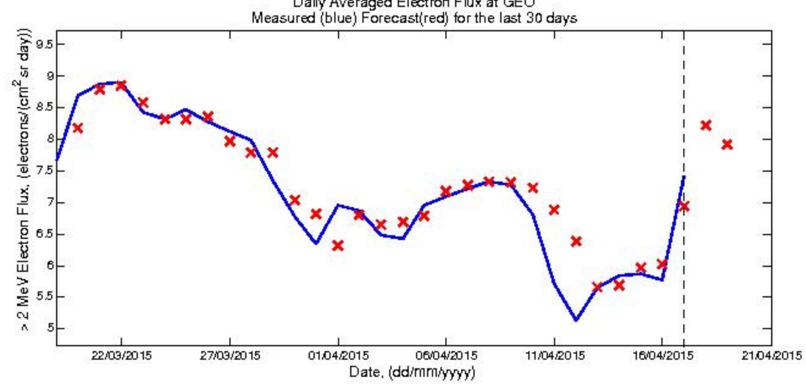
- University of Sheffield
- Finnish Meteorological Institute
- University of Warwick
- Skolkovo Institute of Science and Technology
- University of Michigan
- Space Research Institute, Ukraine
- LPC2E, France
- Swedish Institute for Space Physics

Collaborators

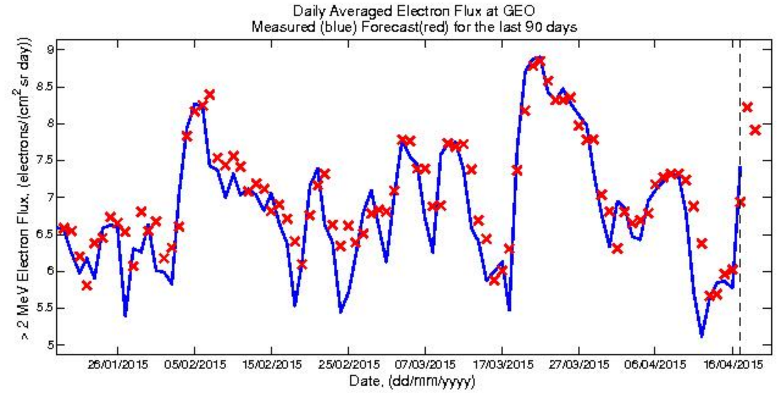
- Berkeley University
- UCLA

PROGRESS has received funding from the *European Union's Horizon 2020* under grant agreement No 637302.

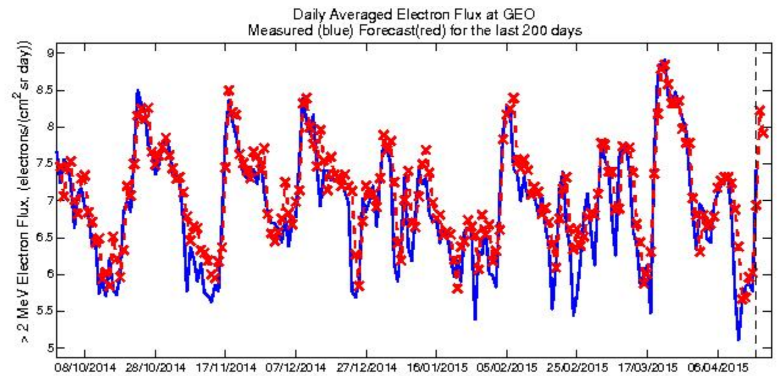
The one day ahead forecasts of the relativistic electron fluxes with energies greater than 2 MeV at GEO has been developed in Sheffield and is available in real time:
http://www.ssg.group.shef.ac.uk/USSW/2MeV_EF.html.
The PE for this model calculated for the period 14 April 2010 and 12 April 2013 is equal to 0.786



Past 90 days



Past 200 days



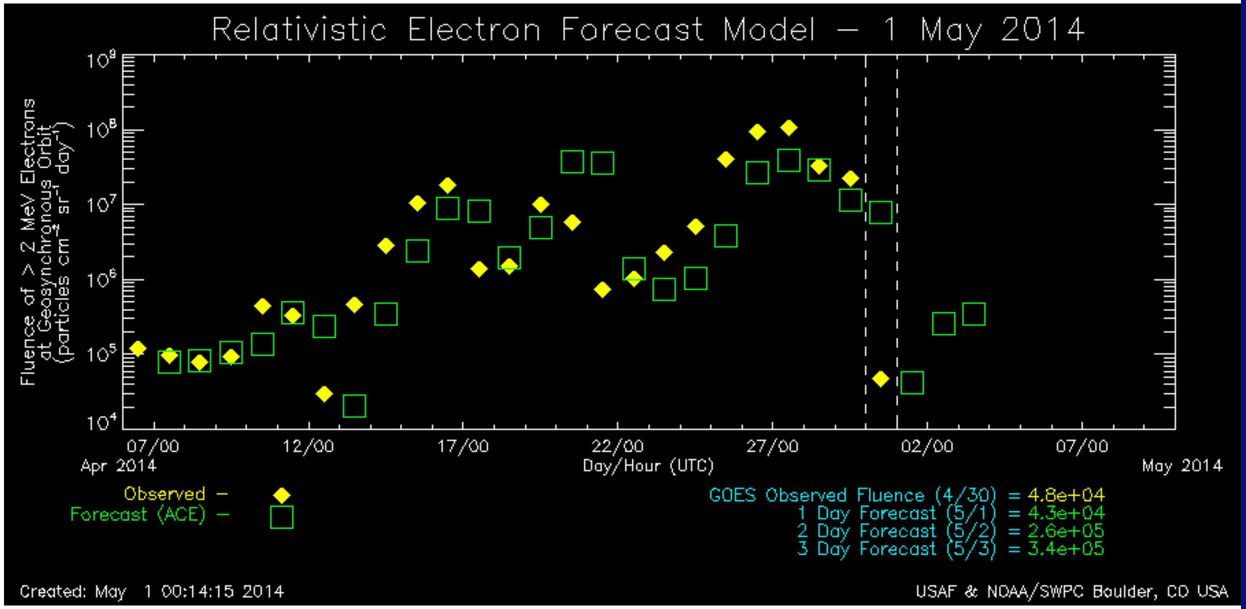
NOAA REFM Forecast

Space Weather Prediction Center 01/05/2014 21:09

NOAA / Space Weather Prediction Center

Relativistic Electron Forecast Model

Presented by the USAF and NOAA/ Space Weather Prediction Center



The impact of high-energy (relativistic) electrons on orbiting satellites can cause electric discharges across internal satellite components, which in turn leads to spacecraft upsets and/or complete satellite failures. The Relativistic Electron Forecast Model predicts the occurrence of these electrons in geosynchronous orbit.
Plots and data are updated daily at 0010 UT. Dashed vertical lines indicate the last vertical value.
When the input parameters are not available, the forecast is not shown.

REFM Verification Plot and Model Documentation

1 to 3 Day Predictions (text file) and corresponding Performance Statistics.
Predictions created using data from the ACE spacecraft.

Historical electron particle data is archived at the
National Geophysical Data Center for Solar-Terrestrial Physics.

Visually impaired users may contact SWPC for assistance.
Please credit SWPC when using these images.

Comparison of REFM and SNB^{3}GEO Forecasts (01.03.2012-03.07.2014)

$$PE = 1 - \frac{1}{N}\sum \frac{(Y(t) - Ym(t))^2}{\mathrm{var}(Y)}$$

$$C_{cor} = \frac{1}{N}\sum \frac{(Y(t) - \langle Y(t)\rangle)(Ym(t) - \langle Ym(t)\rangle)}{\sqrt{\mathrm{var}(Ym)\mathrm{var}(Y)}}$$

Comparison of REFM and SNB^{3}GEO Forecasts

Balikhin, Rodriguez, Boynton, Walker, Sibeck Billings, submitted to SW 2015

Model	Prediction Efficiency Flux	Correlation Flux	Prediction Efficiency Log Flux	Correlation Log Flux
REFM	-1.31	0.73	0.70	0.85
SNB^{3}GEO	0.63	0.82	0.77	0.89

Comparison of REFM and SNB^{3}GEO Forecasts

Balikhin, Rodriguez, Boynton, Walker, Sibeck Billings, submitted to SW 2015

Table 2. Contingency tables and Heidke skill scores for the REFM predictions.

Fluence ($cm^{-2}sr^{-1}day^{-1}$)	$> 10^8$		$> 10^{8.5}$		$> 10^9$	
REFM HSS	0.666		0.482		0.437	
Observation:	Yes	No	Yes	No	Yes	No
Forecast						
Yes	86	22	23	22	4	7
No	43	510	21	595	3	647

Table 3. Contingency tables and Heidke skill scores for the SNB^{3}GEO predictions.

Fluence ($cm^{-2}sr^{-1}day^{-1}$)	$> 10^8$		$> 10^{8.5}$		$> 10^9$	
SNB^{3}GEO HSS	0.738		0.634		0.612	
Observation:	Yes	No	Yes	No	Yes	No
Forecast						
Yes	106	33	31	19	4	2
No	23	499	13	598	3	652

$$S = \frac{2(xw - yz)}{y^2 + z^2 + 2xw + (y+z)(x+w)}$$

"Physical Based Versus Data Based"

For Physical Models:

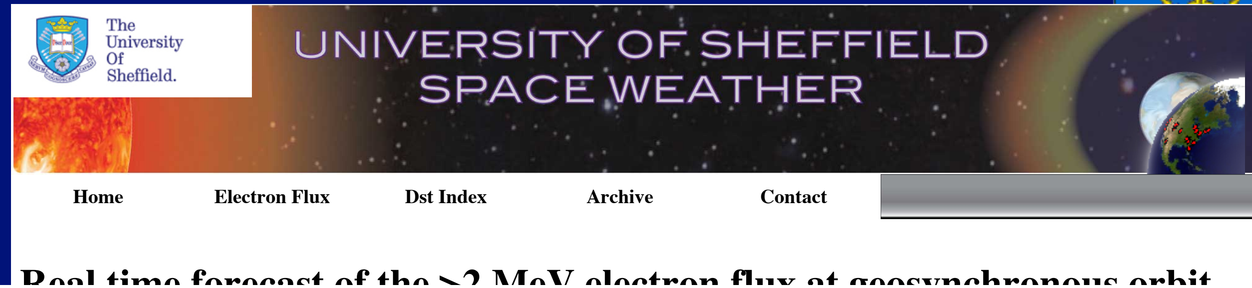
What are the scientific assumptions underpinning the model? (eg MHD formulation, comprehensive or simplified representation of physical processes)

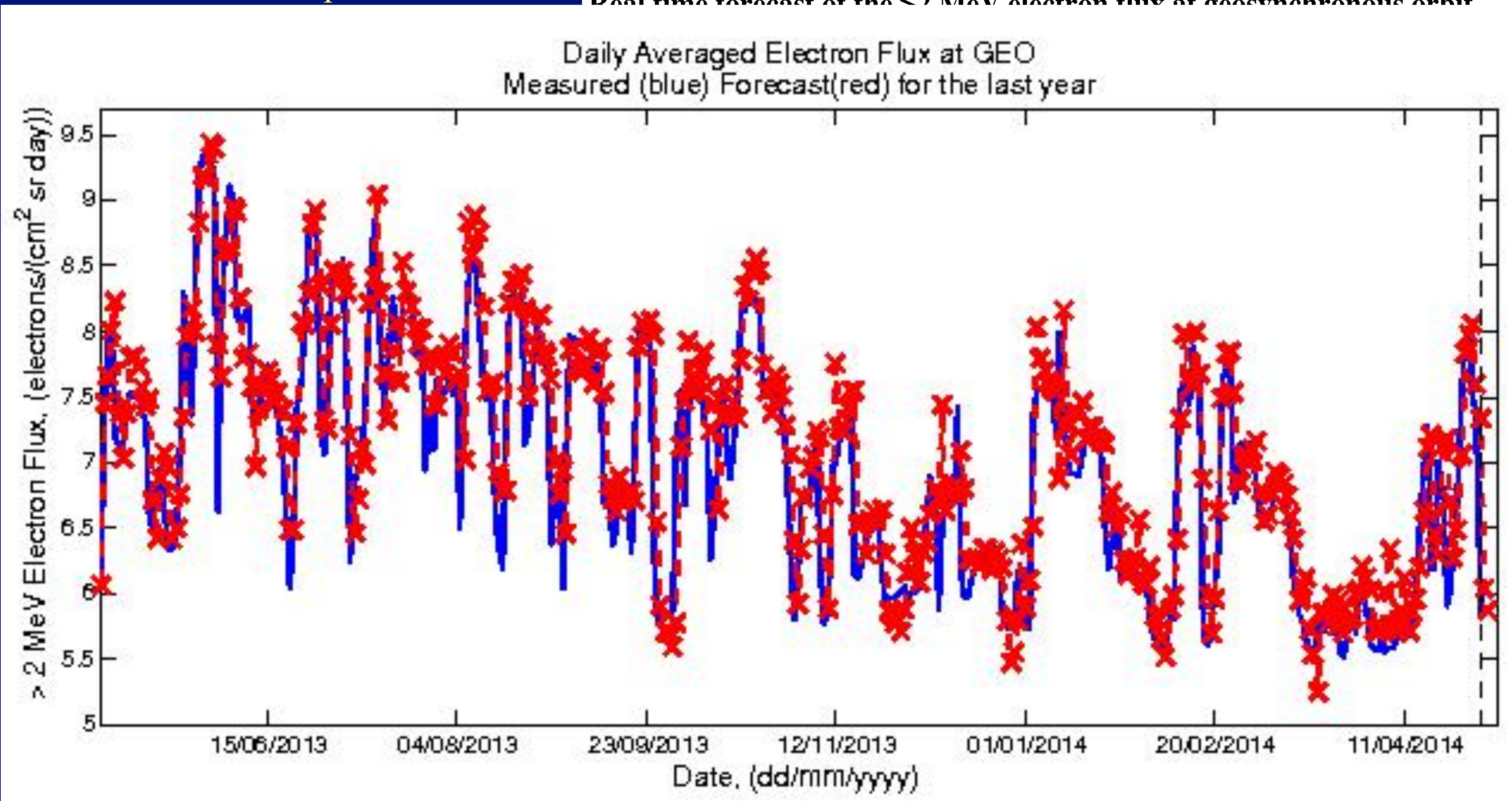
How will the model physics scale to extremes?

For Empirical Models

Empirical models are assumed to be very unlikely to handle extremes as they do not scale – do you agree?

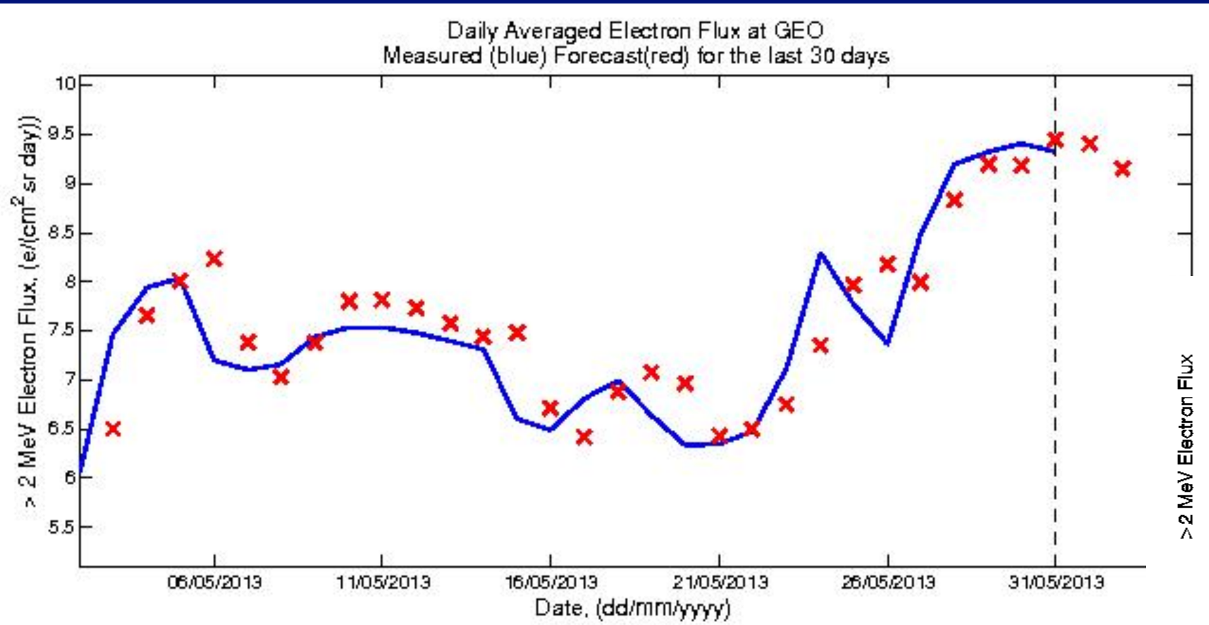
The one day ahead forecasts of the relativistic electron fluxes with energies greater than 2 MeV at GEO has been developed in



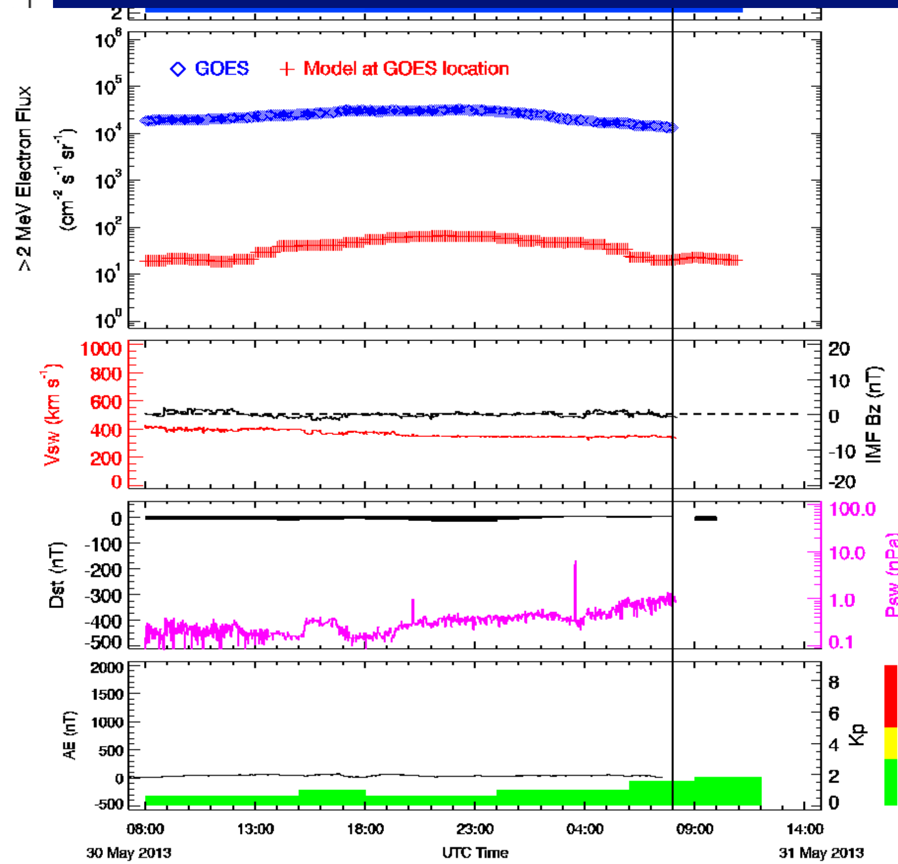


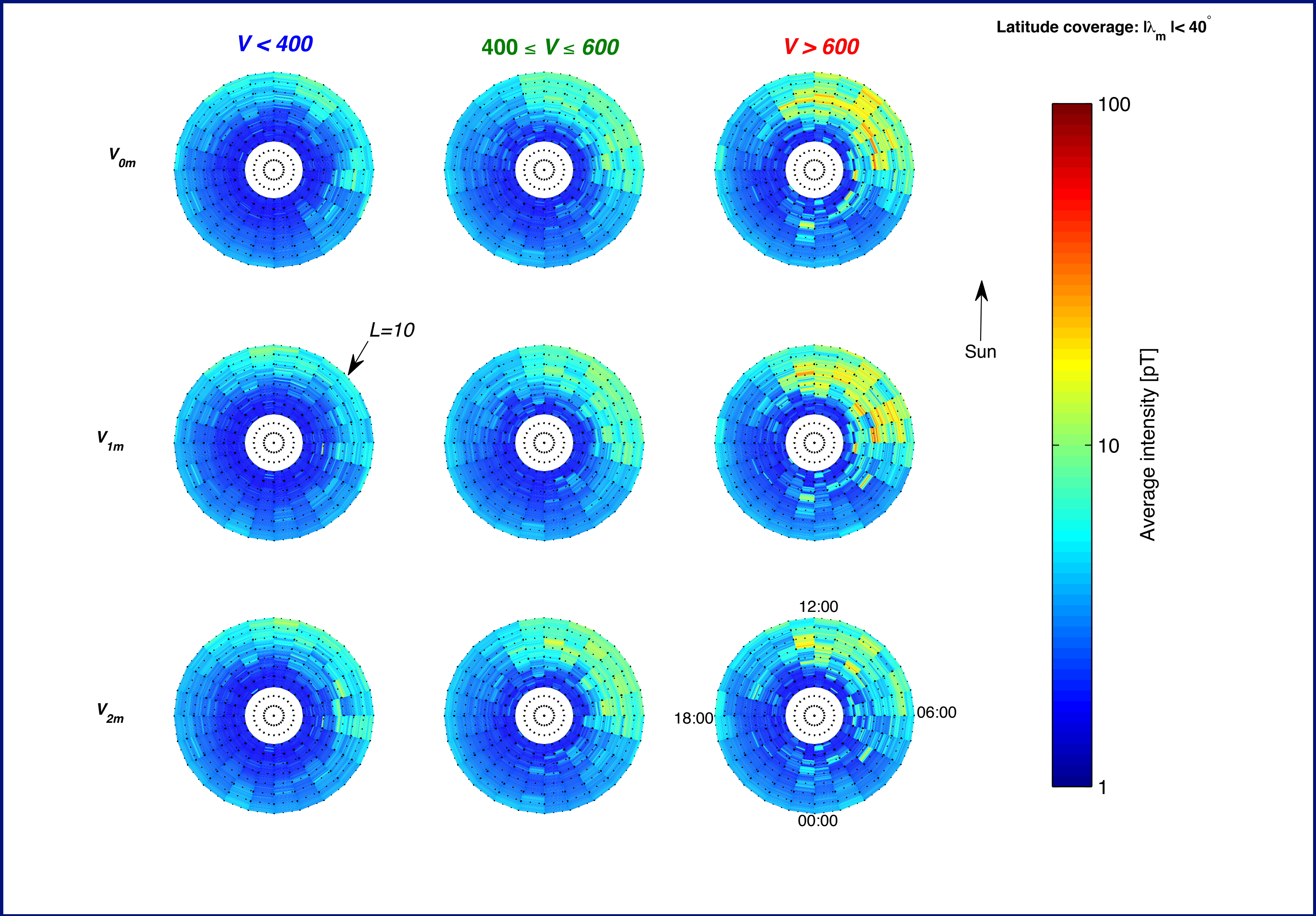
May 29-31 2013

System Science -NARMAX



Forecast From the First Principles





Latitude coverage: |λm |< 40°
V < 400
400 ≤ V ≤ 600
V > 600
V0m
V1m
V2m
L=10
Sun
12:00
18:00
06:00
00:00
100
10
1
Average intensity [pT]

What is referred to as “Physics Approach”

Analytical Approach

$$S = \int L(x,\dot{x},t)\,dt$$

$$dL = \sum_i \frac{\partial L}{\partial x_i} dx_i + \sum_i \frac{\partial L}{\partial \dot{x}_i} d\dot{x}_i$$

Assumptions

+

Physical Knowledge

+

First Principles

What is referred to as “Physics Approach”

Analytical Approach

$$S = \int L(x, \dot{x}, t)\,dt$$

$$dL = \sum_i \frac{\partial L}{\partial x_i} dx_i + \sum_i \frac{\partial L}{\partial \dot{x}_i} d\dot{x}_i$$

Assumptions

+

Physical Knowledge

+

First Principles

Whenever a theory appears to you as the only possible one, take this as a sign that you have neither understood the theory nor the problem which it was intended to solve.

Karl Raimund Popper

Complex Systems

Analytical Approach

$$S = \int L(x, \dot{x}, t)\,dt$$

$$dL = \sum_i \frac{\partial L}{\partial x_i} dx_i + \sum_i \frac{\partial L}{\partial \dot{x}_i} d\dot{x}_i$$

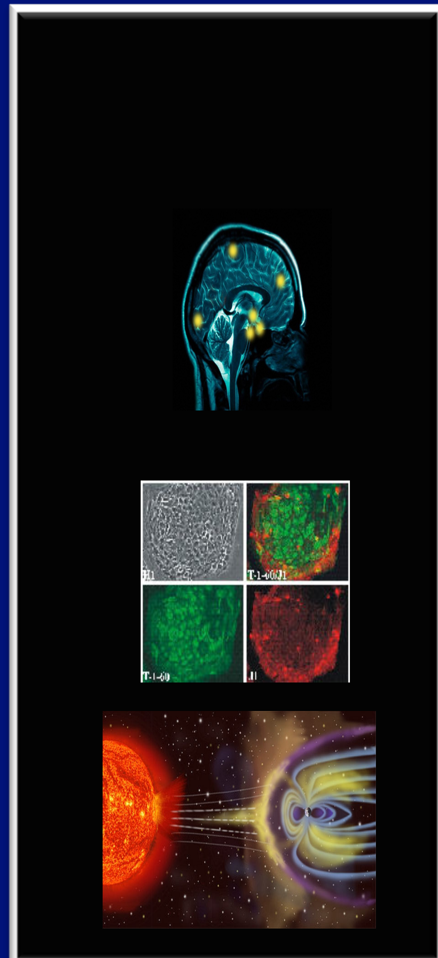
Assumptions

+

Physical Knowledge

+

First Principles



System Identification Approach

Analytical Approach

$$S = \int L(x, \dot{x}, t)\,dt$$

$$dL = \sum_i \frac{\partial L}{\partial x_i} dx_i + \sum_i \frac{\partial L}{\partial \dot{x}_i} d\dot{x}_i$$

Assumptions

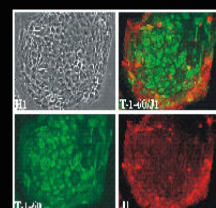
+

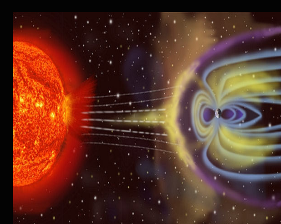
Physical Knowledge

+

First Principles

Black box System



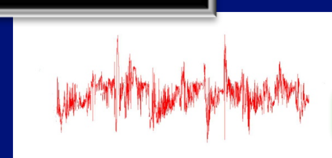


Systems Approach

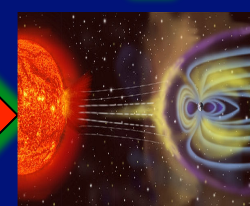
Knowledge of the System

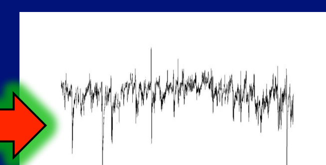
$$S = \int L(x, \dot{x}, t)\,dt$$

$$dL = \sum_i \frac{\partial L}{\partial x_i} dx_i + \sum_i \frac{\partial L}{\partial \dot{x}_i} d\dot{x}_i$$



Input Data

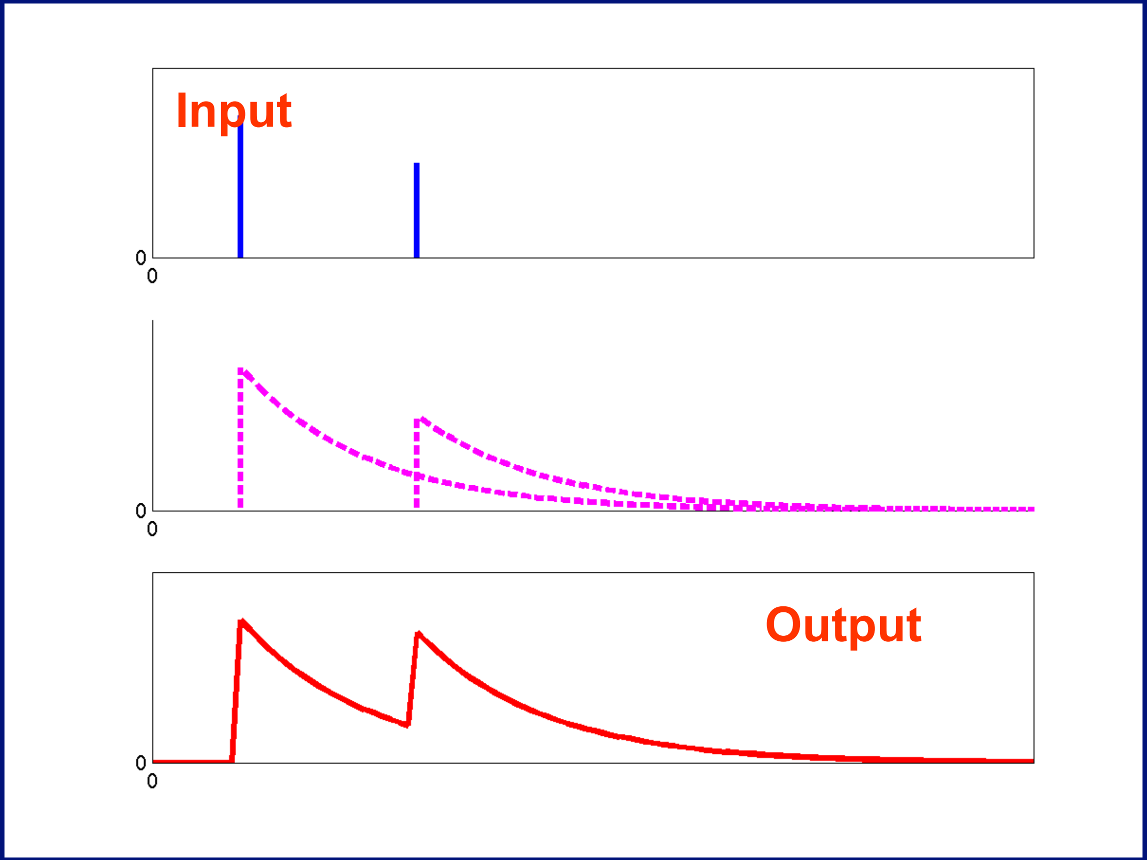




Output Data

Linear System :
(Superposition Principle is valid)

$$x = a_1\delta(t-\tau_1) + a_2\delta(t-\tau_2)$$
$$y(t) = a_1h(t-\tau_1) + a_2h(t-\tau_2)$$

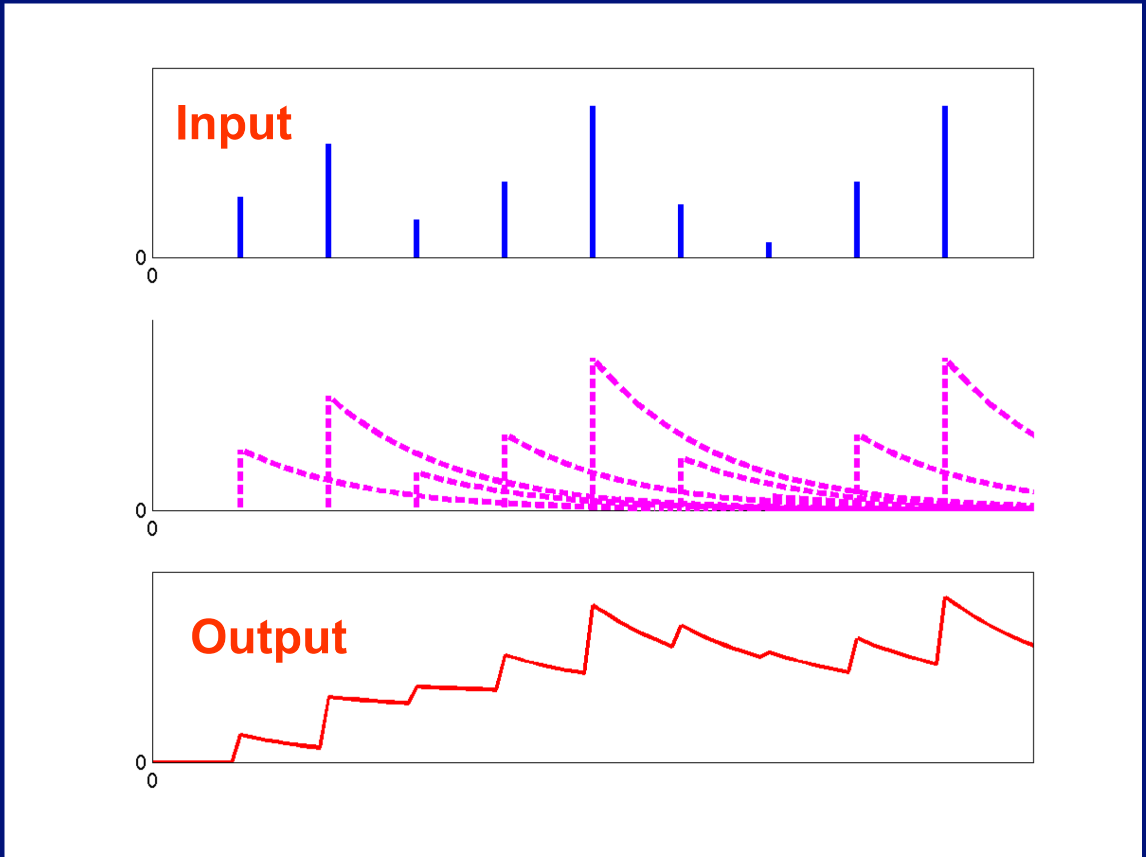


Linear System :
(Superposition Principle is valid)

$$x = \sum a_i \delta(t - \tau_i)$$

$$y(t) = \sum a_i h(t - \tau_i)$$

$$y(t) = \int_0^\infty h(\tau) x(t - \tau) d\tau$$



Time and Frequency domain representation of *Linear System*

Y=D[X] Action of linear black box can be represented either in

the time domain via *Impulse Response Function*:

$$y(t)=\int_0^{\infty} h_1(\tau)x(t-\tau)d\tau$$

Or in the frequency domain via *Linear Frequency Response Function:*

$$Y_f = H_1(f)X_f$$

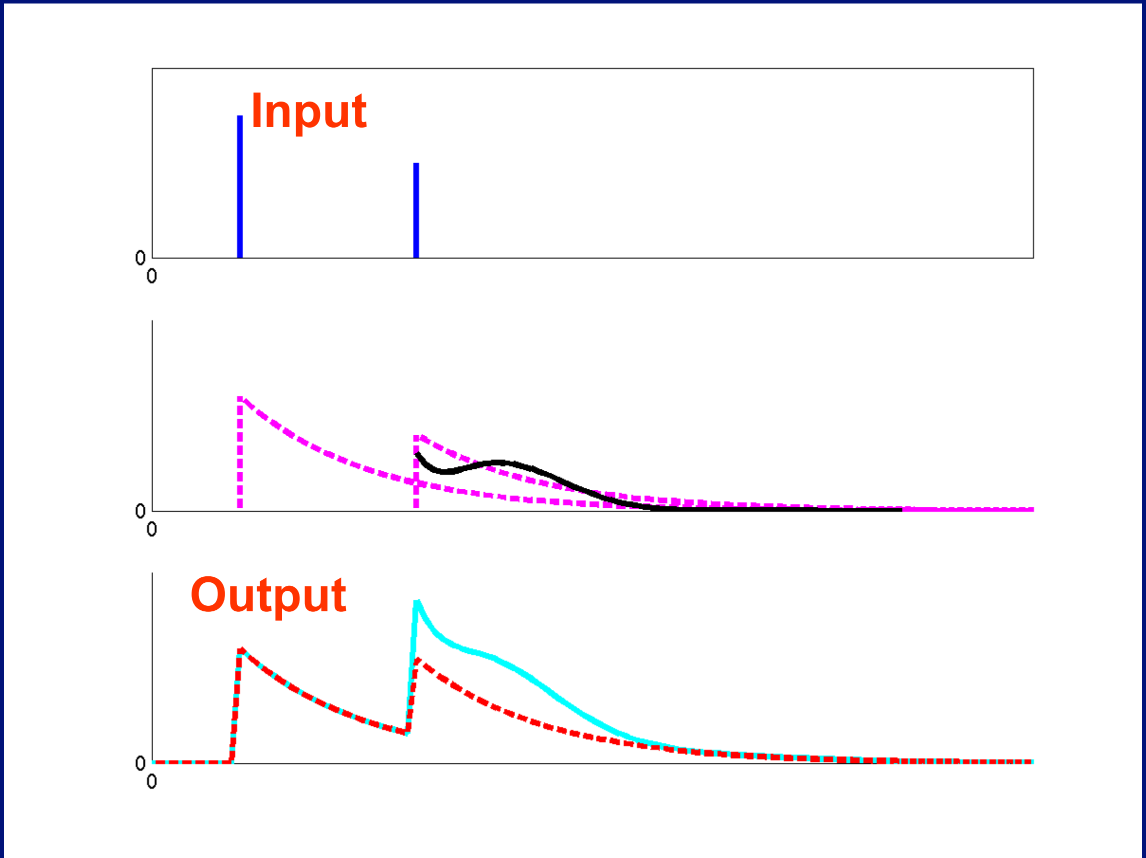
H_1 describes linear amplification (attenuation) of a spectral component and its time delay

Non-linear System

(Superposition Principle is non valid)

$$x = a_1\delta(t-\tau_1) + a_2\delta(t-\tau_2)$$

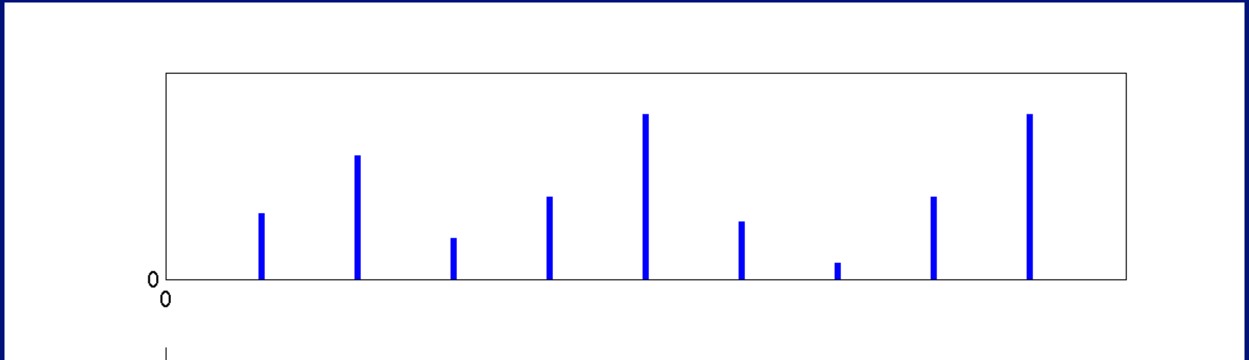
$$y(t) = a_1 h(t-\tau_1) + a_2 h(t-\tau_2) + h_2(\tau_1,\tau_2)a_1 a_2$$



Non-linear System

(Superposition Principle is non valid)

$$y(t) = \sum_{\tau_i} x(t-\tau_i)h(\tau_i) + \sum_{(\tau_i,\tau_j)} x(t-\tau_i)x(t-\tau_j)h_2(\tau_1,\tau_2) + \ldots$$



Non-linear System

(Superposition Principle is non valid)

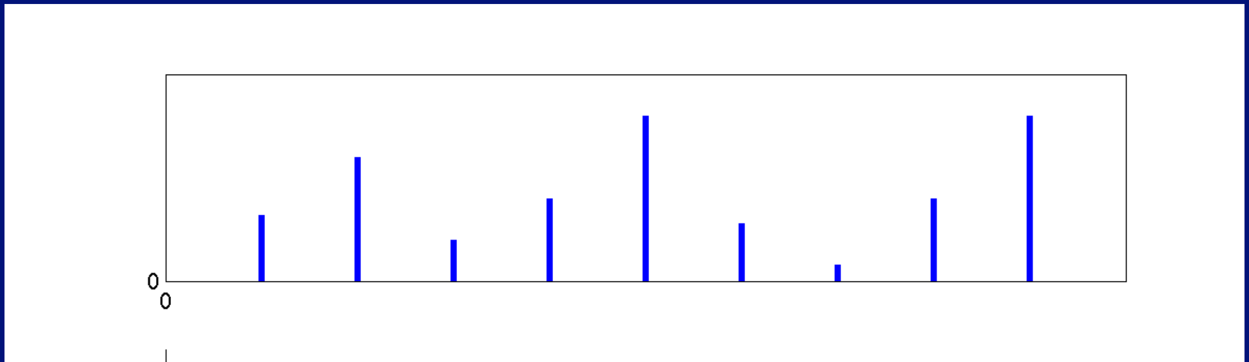
$$y(t) = \sum_{\tau_i} x(t-\tau_i)h(\tau_i) + \sum_{(\tau_i,\tau_j)} x(t-\tau_i)x(t-\tau_j)h_2(\tau_1,\tau_2) +$$

$$\sum_{(\tau_i,\tau_j,\tau_k)} x(t-\tau_i)x(t-\tau_j)x(t-\tau_k)h_3(\tau_1,\tau_2,\tau_k).+\ ....$$

$$y(t) = \int_{-\infty}^{\infty} h_1(\tau)u(t-\tau)d\tau +$$

$$+\iint \quad h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2$$

$$+\iiint \quad h_3(\tau_1,\tau_2,\tau_3)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \ldots$$



Nonlinear Systems:Frequency Domain

Time domain

$$y(k) = \sum_{i=1}^{\infty} h_1(i)\ u(k-i) + \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} h_2(i,j)\ u(k-i)u(k-j) +$$
$$+ \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} h_3(i,j,n)\ u(k-i)u(k-j)u(k-n) + \ldots.$$

N-fold Fourier transform of the nth Volterra kernel George

Frequency domain GFRF

Generalyzed Frequency Response Functions

$$Y_f = H_1(f)X_f + \sum_{f_1+f_2=f} H_2(f_1,f_2)X_{f_1}X_{f_2} + \sum_{f_1+f_2+f_3=f} H_3(f_1,f_2,f_3)X_{f_1}X_{f_2}X_{f_3} + \ldots.$$

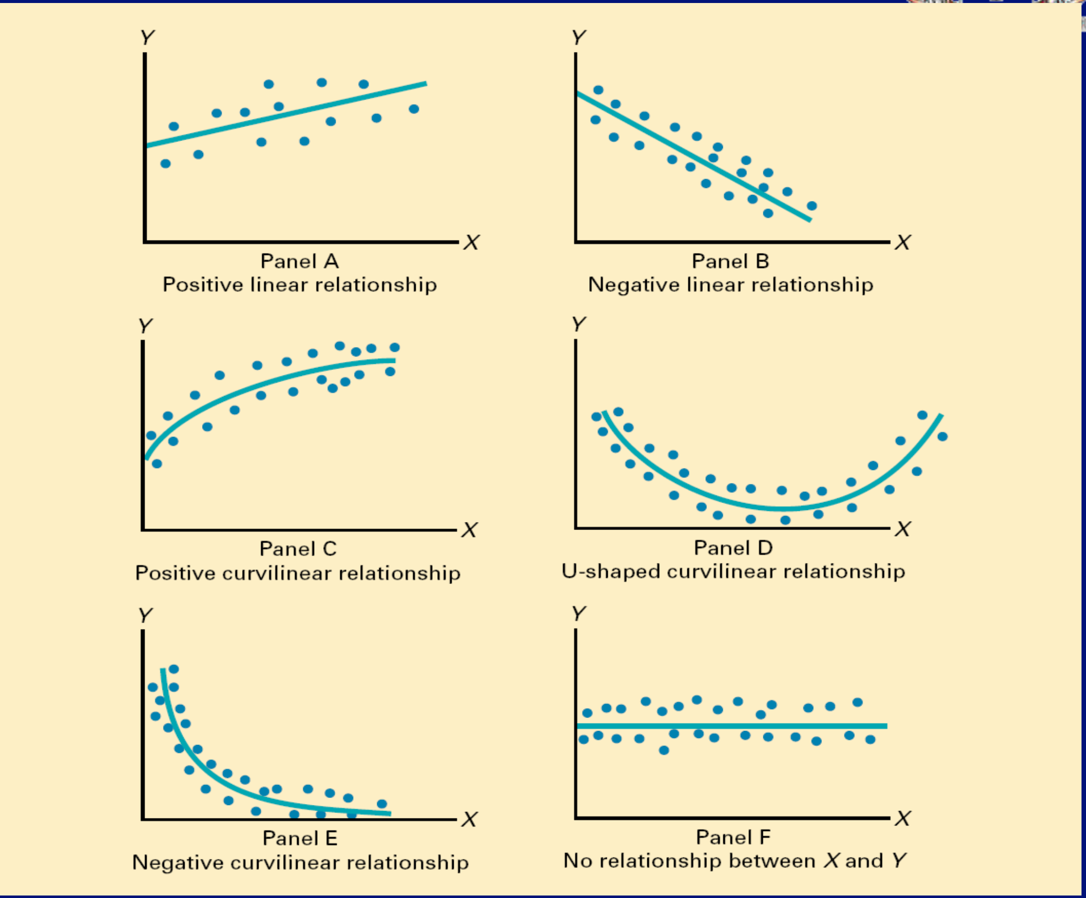
H_1 describes growth (damping) rate of wave and dispersion

H_2 describes 3 wave processes e.g. decay instability

H_3 describes 4 wave processes e.g. modulational instability

Input-Output data sets

If the aim is to develop model that minimises errors many simple methods can be use:d
Fuzzy Logic, Neural Networks , Bayesian metods etc



NARNAX aimed at the simplest model that reproduces the system dynamics, the model that can be related to the components of the system. The model that can be interpretable

Nonlinear AutoRegressive Moving Average model with eXogenous input. NARMAX

$$y(k) = F\left[x, y, \xi\right]$$

x : input; y : output; ξ : noise

Instead of the search for the explicit form of *F*, its decomposition using some basis (e.g. polynomial) is identified.

$$y(k) = F[y(k-1),\dots y(k-n_y),u(k),\dots,u(k-n_u),\xi(k-1),\dots,\xi(k-n_\xi)] + \xi(k)$$

Polynomial expansion of *F*

$$y(k) = \sum_{m=0}^{M} \theta_m p_m(k) + \xi(k)$$

Objective to estimate θ_m

$$y(k) = F[y(k-1),\dots y(k-n_y),u(k),\dots,u(k-n_u),\xi(k-1),\dots,\xi(k-n_\xi)] + \xi(k)$$

Polynomial expansion of *F*

$$y(k) = \sum_{m=0}^{M} \theta_m p_m(k)$$

Even the objective is to estimate θ_m the algorithm is formulated for the auxiliary model:

$$y(k) = \sum_{m=0}^{M} g_m w_m(k)$$

where $w_m(k)$ are constructed to be orthogonal over data so if $j \neq i$

$$\sum_{k=1}^{L} w_j(k) w_i(k) = 0$$

$$\sum_{i=1}^{L} w_j(k) w_i(k) = 0;$$

$$w_1(k) = p_1(k)$$

If W_{m-1} polynomial is known W_m can be found by:

$$w_m(k) = p_m(k) - \sum_{r=0}^{m-1} \alpha_{rm} w_r(k), \ \mathrm{m} = 1,\ldots,\mathrm{M}$$

$$\alpha_{rm} = \frac{\sum_{k=1}^{N} p_m(k) w_r(k)}{\sum_{k=1}^{N} w_r^2(k)}; \qquad 0 \le r \le m\text{-}1$$

Estimation of the auxiliary model

$$y(k)=\sum_{m=0}^{M} g_m w_m(k)$$

$$w_n(k)y(k)=w_n(k)\sum_{m=0}^{M} g_m w_m(k)$$

$$\hat{g}_n=\frac{\sum_{k=1}^{N} w_n(k)y(k)}{\sum_{k=1}^{N} w_n^2(k)}$$

From the auxiliary model to NARMAX model

$$y(k)=\sum_{m=0}^{M} g_m w_m(k) \quad \Rightarrow \quad y(k)=\sum_{m=0}^{M} \theta_m p_m(k)$$

Solar Wind Magnetosphere "Coupling Functions"

Name	Functional Form	Reference
B_z	B_z	*Dungey* [1961]
Velocity	v	*Crooker et al.* [1977]
Density	n	
p	$nv^2/2$	*Chapman and Ferraro* [1931]
Bs	B_z $(B_z < 0)$; 0 $(B_z > 0)$	
Half-wave rectifier	vBs	*Burton et al.* [1975]
ε	$vB^2\sin^4(\theta_c/2)$	*Perrault and Akasofu* [1978]
ε_2	$vB_T^2\sin^4(\theta_c/2)$	Variant on ε
ε_3	$vB\sin^4(\theta_c/2)$	Variant on ε
Solar wind E-field	vB_T	
E_{KL}	$vB_T\sin^2(\theta_c/2)$	*Kan and Lee* [1979]
$E_{KL}{}^{1/2}$	$[vB_T\sin^2(\theta_c/2)]^{1/2}$	Variant on the Kan-Lee electric field
E_{KLV}	$v^{4/3}B_T\sin^2(\theta_c/2)p^{1/6}$	*Vasyliunas et al.* [1982]
E_{WAV}	$vB_T\sin^4(\theta_c/2)$	*Wygant et al.* [1983]
$E_{WAV}{}^2$	$[vB_T\sin^4(\theta_c/2)]^2$	Variant on E_{WAV}
$E_{WAV}{}^{1/2}$	$[vB_T\sin^4(\theta_c/2)]^{1/2}$	Variant on E_{WAV}
E_{WV}	$v^{4/3}B_T\sin^4(\theta_c/2)p^{1/6}$	*Vasyliunas et al.* [1982]
E_{SR}	$vB_T\sin^4(\theta_c/2)p^{1/2}$	*Scurry and Russell* [1991]
E_{TL}	$n^{1/2}v^2B_T\sin^6(\theta_c/2)$	*Temerin and Li* [2006]
$d\Phi_{MP}/dt$	$v^{4/3}B_T{}^{2/3}\sin^{8/3}(\theta_c/2)$	This paper

From Newell et al., 2007

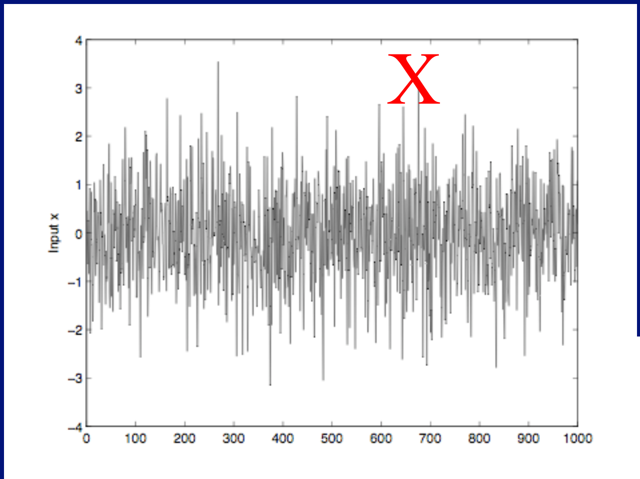
Table 2. Various Possible Viscous Solar Wind Coupling Functions, Ranked According to Their Ability to Predict Variance in 10 Magnetospheric State Variables

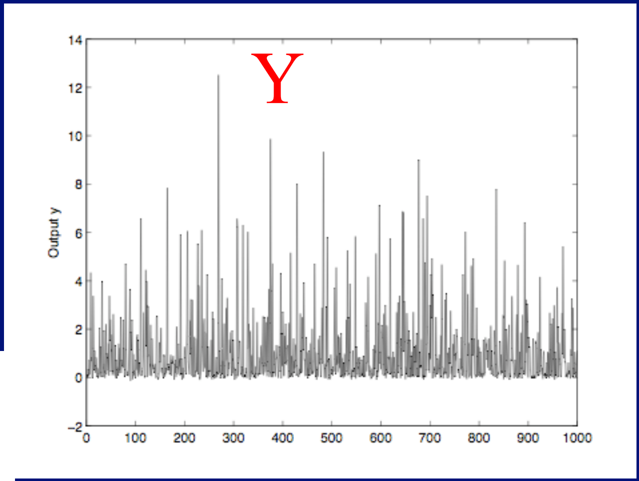
Rank, f	Λ_c	Dst	AE	AU	Goes	Kp	Auro	b2i	Φ_{PC}	AL	$\Sigma r^2/n$
1. $n^{1/2}v^2$	−0.364	−0.500	0.469	0.430	−0.325	0.670	0.510	−0.520	0.319	−0.225	22.3%
2. $n^{1/3}v^2$	−0.371	−0.497	0.458	0.389	−0.353	0.678	0.512	−0.460	0.324	−0.250	21.8%
3. $n^{1/2}v^3$	−0.363	−0.517	0.452	0.383	−0.340	0.653	0.515	−0.449	0.317	−0.236	21.1%
4. $n^{1/6}v^2$	−0.353	−0.460	0.416	0.330	−0.347	0.628	0.471	−0.382	0.294	−0.254	18.5%
5. nv^3	−0.331	−0.507	0.425	0.421	−0.260	0.549	0.488	−0.516	0.272	−0.153	18.5%
6. $nv^{5/2}$	−0.312	−0.457	0.383	0.401	−0.239	0.525	0.448	−0.511	0.249	−0.124	16.3%
7. $v^{4/3}$	−0.374	−0.408	0.372	0.277	−0.321	0.547	0.402	−0.314	0.252	−0.250	14.7%
8. v	−0.324	−0.406	0.374	0.279	−0.321	0.537	0.399	−0.315	0.254	−0.251	14.7%
9. $v^{3/2}$	−0.321	−0.408	0.372	0.276	−0.319	0.549	0.404	−0.312	0.251	−0.249	14.7%
10. v^2	−0.317	−0.409	0.369	0.272	−0.311	0.547	0.407	−0.310	0.247	−0.246	14.4%
11. $v^{2/3}$	−0.325	−0.405	0.374	0.281	−0.311	0.503	0.396	−0.316	0.255	−0.252	14.4%
12. $v^{1/2}$	−0.325	−0.403	0.374	0.282	−0.294	0.465	0.395	−0.316	0.255	−0.252	14.0%
13. p	−0.277	−0.373	0.316	0.357	−0.202	0.469	0.391	−0.474	0.217	−0.085	12.5%
14. $p^{2/3}$	−0.272	−0.321	0.326	0.365	−0.199	0.486	0.377	−0.485	0.228	−0.101	12.4%
15. $p^{1/2}$	−0.267	−0.295	0.329	0.367	−0.194	0.482	0.366	−0.486	0.231	−0.108	12.2%
16. $p^{1/3}$	−0.193	−0.269	0.331	0.366	−0.186	0.463	0.353	−0.485	0.231	−0.115	11.7%
17. $p^{3/2}$	−0.274	−0.427	0.288	0.331	−0.183	0.394	0.397	−0.431	0.190	−0.057	11.1%
18. p^2	−0.257	−0.420	0.250	0.292	−0.150	0.288	0.387	−0.351	0.159	−0.031	8.5%
19. nv	−0.163	−0.149	0.143	0.221	−0.089	0.287	0.253	−0.325	0.136	0.004	4.0%
20. n	−0.041	0.030	0.001	0.093	0.033	0.103	0.122	−0.172	0.058	0.070	0.6%

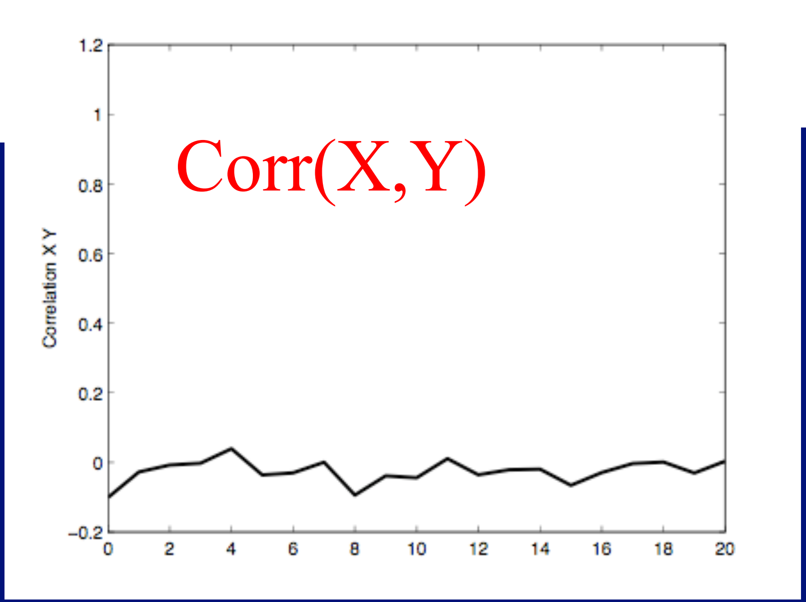
Solar Wind Magnetosphere “Coupling Functions”

X(t) ⇒ x^2 ⇒ Y(t)

5% Noise







(X, Y)

(X^2, Y)

. . . .

(X^k, Y)

(X^{k+1}, Y)

. . . .

$$y(t)=x(t)^2+0.5x(t-1)^4+y(t-1)\,x(t-1)$$

Previously proposed coupling functions

1. $I_B = VB_s$ by *Burton et al.* [1975]
2. $\varepsilon = VB^2 \sin^4(\theta/2)$, by *Perreault and Akasofu* [1978]
3. $I_W = VB_T \sin^4(\theta/2)$ by *Wygant et al.* [1983]
4. $I_{SR} = p^{1/2} VB_T \sin^4(\theta/2)$ by *Scurry and Russell* [1991]
5. $I_{TL} = p^{1/2} VB_T \sin^6(\theta/2)$ by *Temerin and Li* [2006]
6. $I_N = V^{4/3} B_T^{2/3} \sin^{8/3}(\theta/2)$ by *Newell et al.* [2007]
7. $I_V = n^{1/6} V^{4/3} B_T \sin^4(\theta/2)$ by *Vasyliunas et al.* [1982]

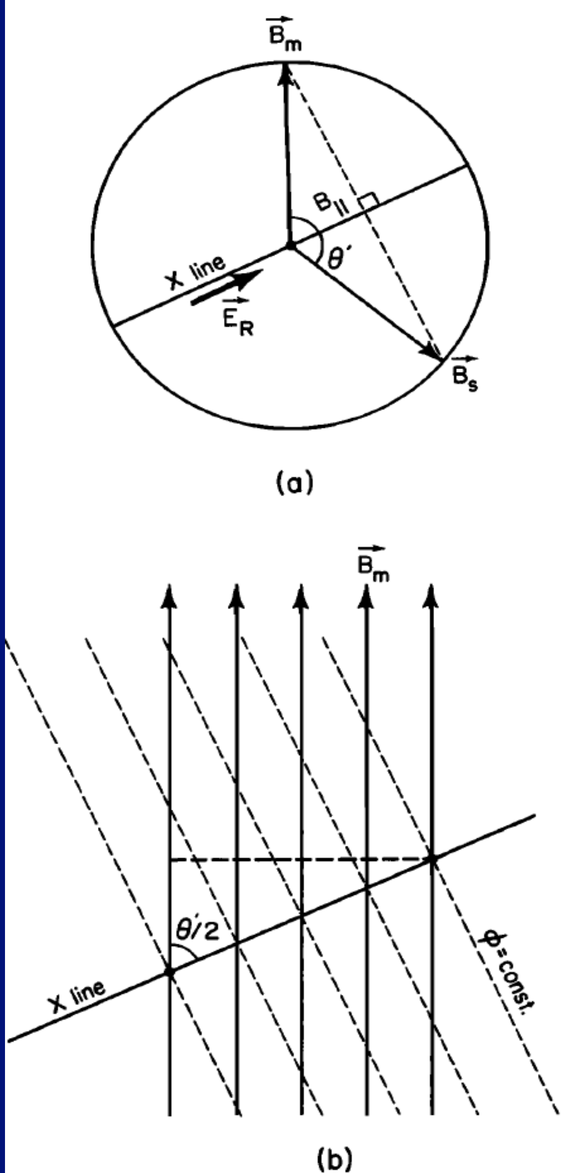
Coupling Function	NERR
$p^{1/2} VB_T \sin^6(\theta/2)(t\text{-}1)$	31.32
$VB_s(t\text{-}1)$	12.76
$n^{1/6} V^{4/3} B_T \sin^4(\theta/2)(t\text{-}1)$	10.30
$p^{1/2} VB_T \sin^4(\theta/2)(t\text{-}1)$	8.37
$D_{st}(t\text{-}2)$	7.23

$p^{1/2}V^{2}B_{T}\sin^{6}(\theta/2)$	14.0
$p^{1/2}V^{4/3}B_{T}\sin^{6}(\theta/2)$	12.5
$P^{1/2}VB_{T}\sin^{6}(\theta/2)$	12.1
VB_{s}	8.91

$\sin^{6}(\theta/2)$ or $\sin^{4}(\theta/2)$?

Where $\sin^{4}(\theta/2)$ did appear from?

Kan and Lee (1978) model



ig. 1. A schematic illustration of the field

$$E_R = V_s B_s \sin\left(\frac{\theta}{2}\right)$$

Reconnection Electric field for two magnetic fields of equal magnitudes: Sonnerup (1974) Russell and Atkinson (1973)

Kan and Lee stated that only perpendicular component of the electric field contributes to the potential across the polar

$$\Phi = \int E_{R\perp} dl_\perp = \int V_s B_s \sin^2\left(\frac{\theta}{2}\right) dl \sin\left(\frac{\theta}{2}\right)$$

$$\Phi = V_s B_s \sin^3\left(\frac{\theta}{2}\right) l_0$$

Finally Kan and Lee argued that power delivered by solar wind dynamo is proportional to potential square divided effective system resistance:

$$P = \frac{\Phi^2}{R} = V_s^2 B_s^2 \sin^6\left(\frac{\theta}{2}\right) l_0^2$$

ENERGY COUPLING FUNCTION AND SOLAR WIND–MAGNETOSPHERE DYNAMO

J.R. Kan and L.C. Lee

The potential difference Φ_m across the polar cap is due to the perpendicular component of the reconnection electric field, i.e., $E_R \sin \theta/2$ as shown in Figure 1(b). This geometrical factor has been overlooked in the previous studies of component reconnection. Thus the polar cap potential Φ_m can be written as

$$\Phi_m = V_s B_s \sin^2 (\theta/2)\, \ell_o \qquad (3)$$

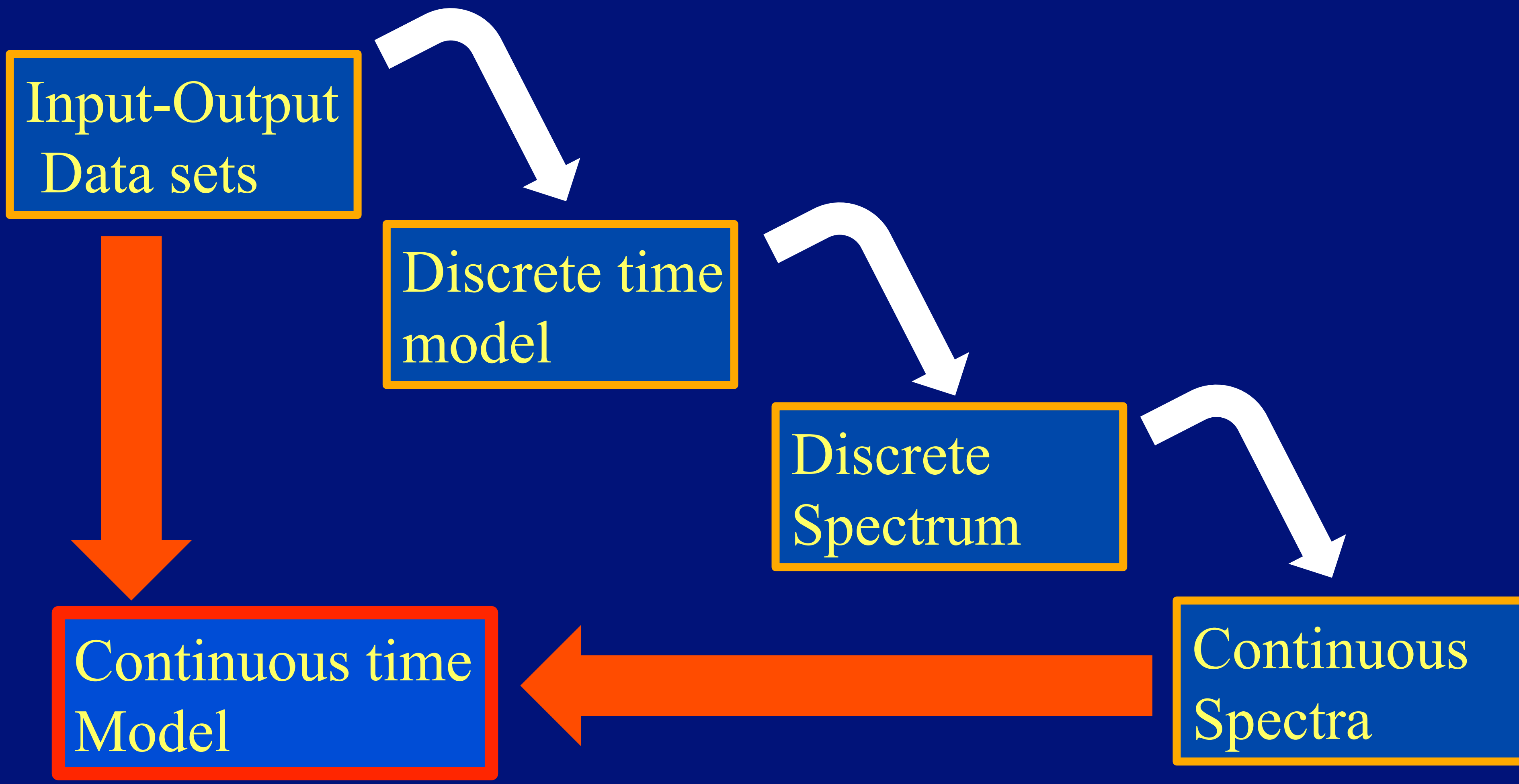
where ℓ_o is the effective length of the X line.

The power delivered by the solar wind dynamo is given by

$$P = \phi_m^2/R = V^2 B^2 \sin^4 (\theta/2)\, \ell_o{}^2/R$$

$$= (V/R)\ \varepsilon\,(t) \qquad (5)$$

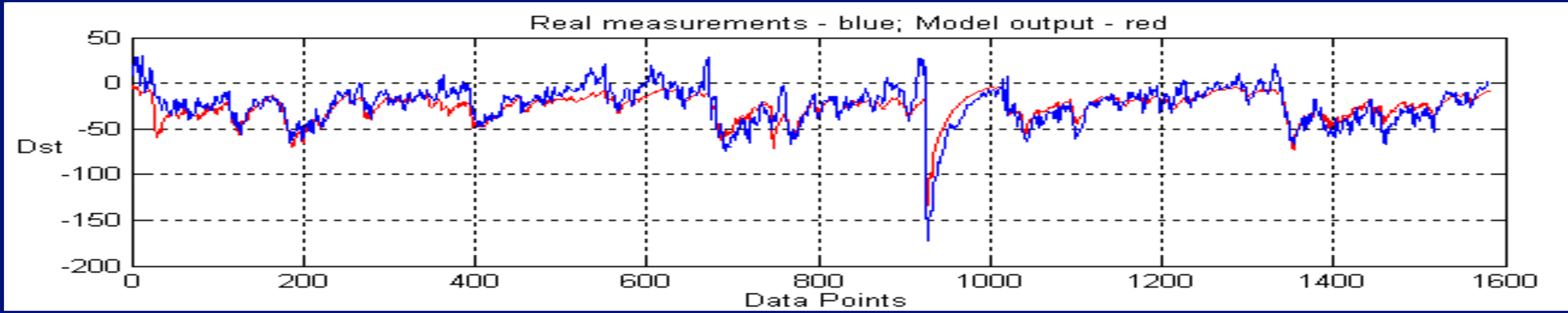
Approach to the derivation of continuous analytical model



Reconstruction of Nonlinear Continuous Time Models

$$D_{st}(t) + 19.282\frac{dD_{st}(t)}{dt} + 5.319VB_s(t) + 1.364\frac{dVB_s(t)}{dt}$$
$$+ 0.104VB_s^{\,2}(t) + 0.0075D_{st}(t)VB_s(t) = 0$$

Forecasting D_{st} with Continuous Time Model (1)



Analysis in the frequency domain
Second order transfer function $H_2(f_1, f_2)$

- Dominant ridge-like maximum: $f_1 + f_2 \to 0$ Energy storage

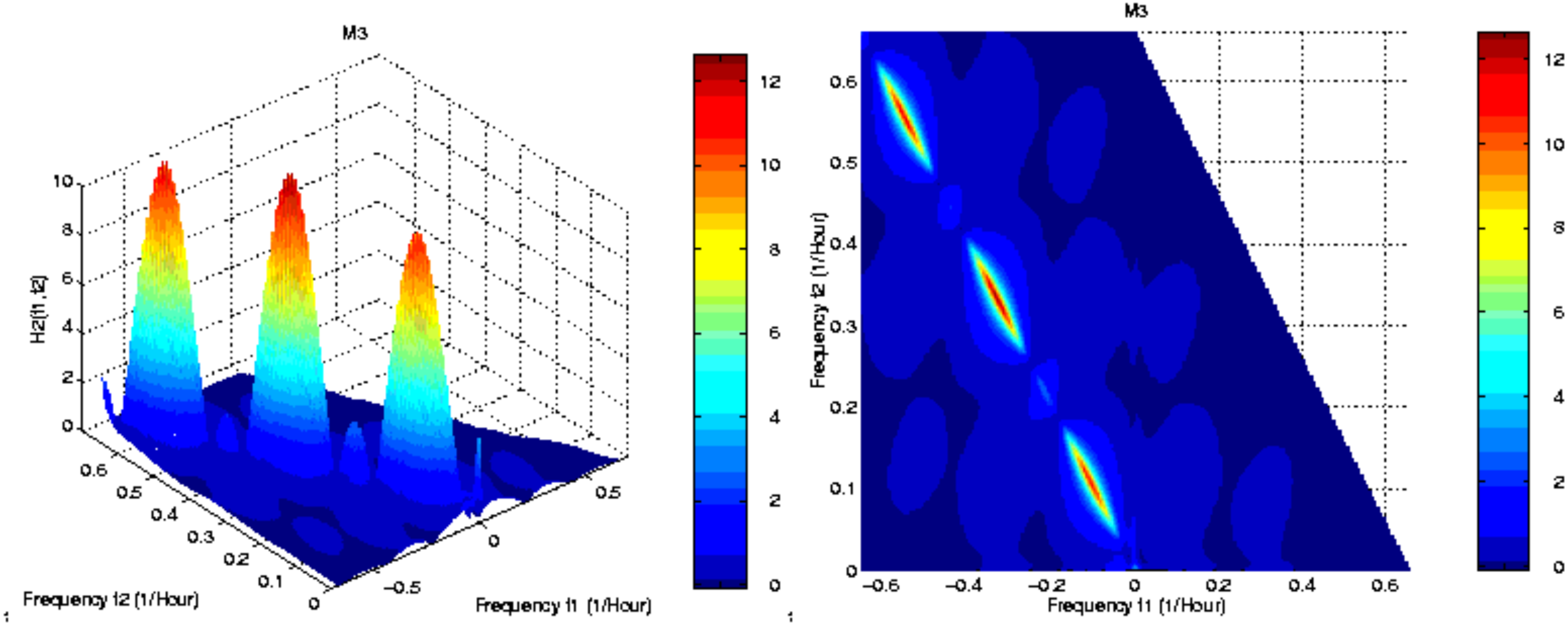


Figure 1:

Analysis in the frequency domain
Second order transfer function $H_2(f_1, f_2)$

- Dominant ridge-like maximum:

$$f_1 + f_2 \rightarrow 0$$ Energy storage

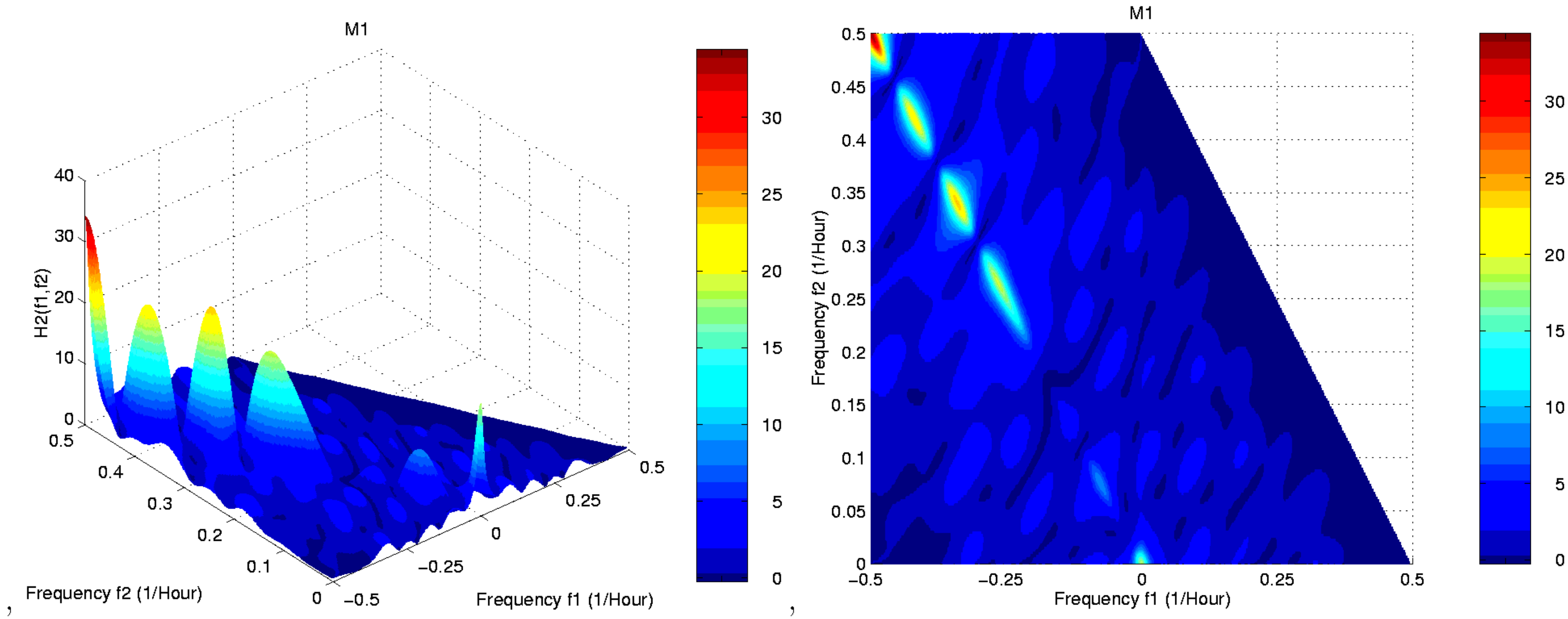


Figure 1:

The magnitude of H_2. Ridge-like maximum coresponds to $f_1 + f_2 = 0$.

$$D_{st}(t) + 19.282\frac{dD_{st}(t)}{dt} + 5.319VB_s(t) + 1.364\frac{dVB_s(t)}{dt}$$
$$+ 0.104VB_s^{\,2}(t) + 0.0075D_{st}(t)VB_s(t) = 0$$

In the absence of the input:

$$D_{st}(t) + 19.282\frac{dD_{st}(t)}{dt} = 0$$

$$D_{st}(t) = D_{st}(0)\exp\left(-\frac{t}{19.282(hours)}\right)$$

The decay time of the model in the absence of the input is independent both upon D_{st} and VB_s

$$D_{st}(t) + 19.282\frac{dD_{st}(t)}{dt} + 5.319VB_s(t) + 1.364\frac{dVB_s(t)}{dt} + 0.104VB_s^{\,2}(t) + 0.0075D_{st}(t)VB_s(t) = 0$$

Solution of the non-homogeneous equation

$$D_{st}(t) = D_{st}(0)\exp\left(-\frac{\int_0^t \left(1 + 0.0075VB_s(t')\right)dt'}{19.282}\right) +$$

$$\int_0^t \left[0.27VB_s(t') + 0.073\frac{dVB_s(t')}{dt} + 0.0054VB_s^{\,2}(t')\right]\left(\exp\frac{\int_0^{t'} \left(1 + 0.0075VB_s(t'')\right)dt''}{19.282}\right)dt' \times$$

$$\exp\left(-\frac{\int_0^t \left(1 + 0.0075VB_s(t')\right)dt'}{19.282}\right)$$

Burton, McPheron, Russell model

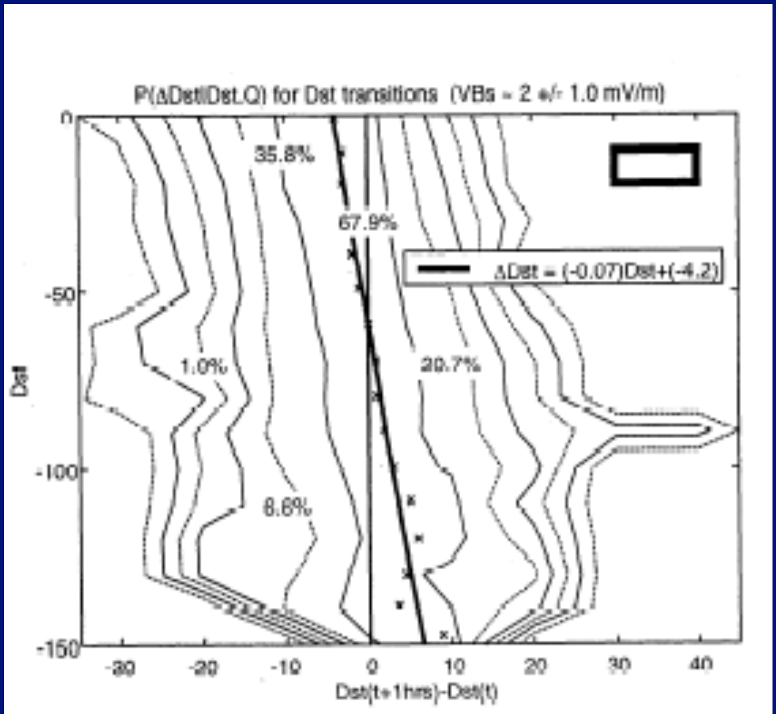
$$\frac{dD_{st}}{dt} = Q(VB_s) - \frac{D_{st}}{\tau}$$

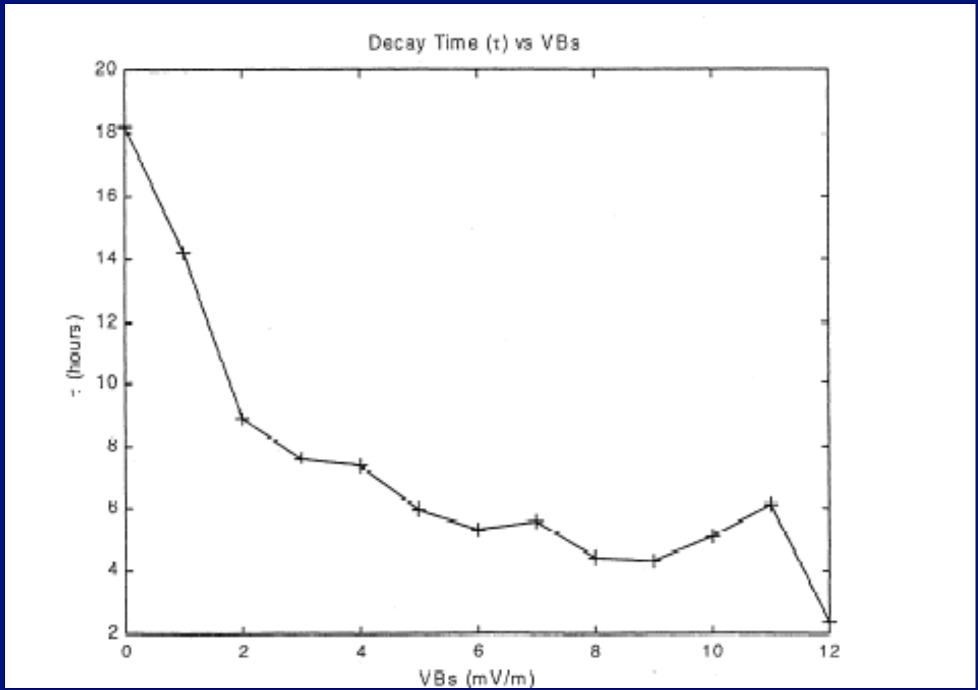
Decay time τ-VBs, O'Brien, McPheron, 2000

$$\frac{dD_{st}}{dt} = Q(VB_s) - \frac{D_{st}}{\tau(VB_s)}$$

$$\Delta D_{st} = D_{st}(t+1) - D_{st}(t) = \left[Q(t) - \frac{D_{st}(t)}{\tau} \right] \Delta t$$

Decay time τ-VBs, O'Brien, McPheron, 2000



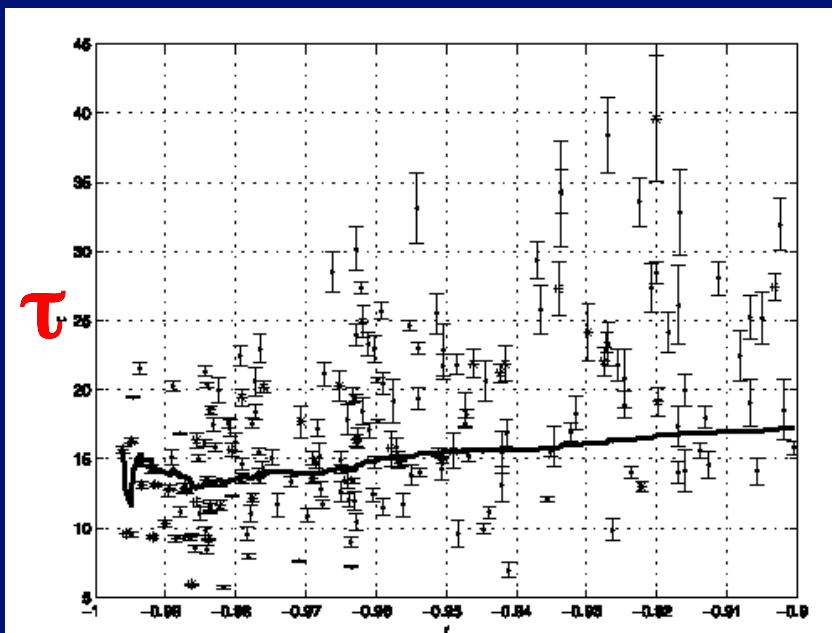


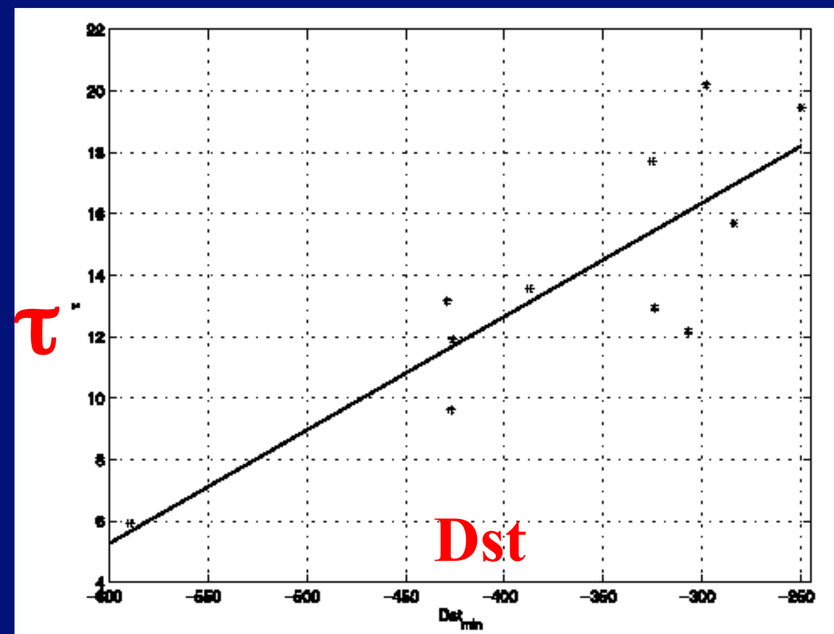
$$\tau \approx 2.4 \exp\left(\frac{9.74}{4.69 + VB_s}\right)$$

1st order Markov approach:

Decay time τ-Dst relation, (Dasso, et al., JGR, 2002).

Assumption: Once the decay phase starts, energy injection is negligible





Main Conclusions: For intense storms the values of τ decrease with the intensity of the storm.

Discussion

1. $\tau \approx 2.4\exp\left(\dfrac{9.74}{4.69+VB_s}\right)$ O'Brien, McPherron., JGR, 7707,2000.

In the case of 'no input' this estimate leads to: $\tau \approx 19.14$

2. Dasso et al., (JGR, 10,1029,2002) have shown *that* D_{st} decay *times* have values between about 5 and 25 hours. Stronger storms exibit shorter decay *time*. As stronger storms assume higher *value* of $\int_0^t VB_s(t)dt$,

$$\tau \approx \frac{19.282(hours)}{1+\dfrac{0.0075}{t}\displaystyle\int_0^t VB_s(t)dt}$$

Conclusion:

1) Whenever a theory appears to you as the only possible one, take this as a sign that you have neither understood the theory nor the problem which it was intended to solve. *(KP)*

2) Data are the main source progress and advanced data analysis technique is important tool not only in temporal validation of hypotheses but also to *falsify*=nullify them.