Comparison of Dst Index Coupling Functions (78323)

H. Al-Saadi and R. J. Boynton, University of Sheffield

Abstract

Local linear filter and nonlinear autoregressive with eXogenous input based on neural network (NARX) were both used to mathematically model and forecast Dst index from input-output data. Several previously proposed solar wind magnetosphere coupling functions were used as input and Dst index was used as output producing two models for each of them. The correlation coefficient and prediction efficiency were used as a means to validate the results. Results from both methods showed that the model employing the Boynton et al [2011] solar wind magnetosphere coupling function produced the best forecast for Dst.

Solar wind-magnetosphere coupling functions

Coupling Function	Reference
$p^{1/2}v^{4/3}B_T\sin^6\left(\frac{\theta}{2}\right)$	Boynton et al., [2011]
vB_s	Burton et al., [1975]
$vB_T\sin^2\left(\frac{\theta}{2}\right)$	Kan and Lee [1979]
$vB_T^2\sin^4\left(\frac{\theta}{2}\right)$	Perreault and Akasofu [1978]
$p^{1/2}vB_T\sin^4\left(\frac{\theta}{2}\right)$	Scurry and Russell [1991]

Model Performance Assessment

In order to assess the produced models performance and compare their forecasts, two methods were used:

Prediction Efficiency

$$PE = 1 - \frac{\sum_{t=1}^{N}\left[\left(y(t)-\hat{y}(t)\right)^2\right]}{\sum_{t=1}^{N}\left[\left(y(t)-\bar{y}(t)\right)^2\right]}$$

Correlation coefficient

$$CC = \frac{\sum_{t=1}^{N}\left[\left(y(t)-\bar{y}(t)\right)\left(\hat{y}(t)-\bar{\hat{y}}(t)\right)\right]}{\sqrt{\sum_{t=1}^{N}\left[\left(y(t)-\bar{y}(t)\right)^2\right]\sum_{t=1}^{N}\left[\left(\hat{y}(t)-\bar{\hat{y}}(t)\right)^2\right]}}$$

Where $y(t)$ is the measured output at time t, $\hat{y}$ is the model predicted output, N is the length of the data and the bar indicates the mean.

Local Linear filters

The local linear filter introduced by Bargatzen et al [1985] can be expressed as autoregressive with eXogenous input model as the following:

$$\hat{y}(t) = a_1\hat{y}(t-1) + \ldots + a_{n_y}\hat{y}(t-n_y) + \ldots$$
$$b_1u(t-1) + \ldots + b_{n_u}u(t-n_{u_1}) + e(t)$$

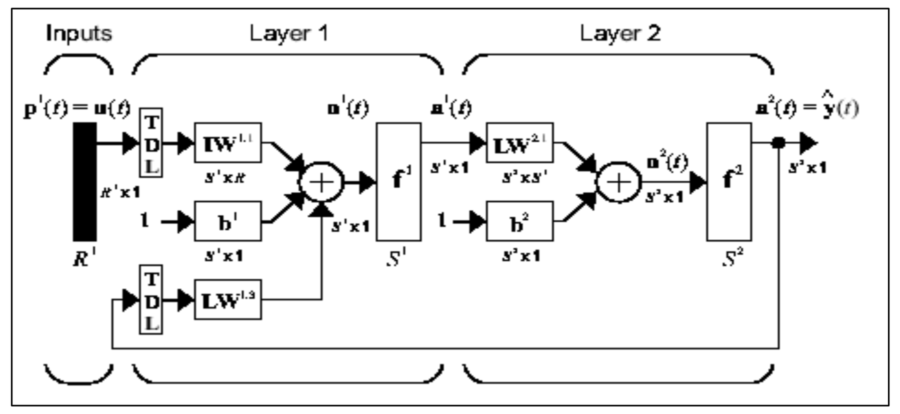
Where $\hat{y}(t)$ is the estimated output at time t, u is the input, n are the maximum lags, and a and b are coefficients.

For each of the hourly averaged coupling functions, the maximum lags were set to 2 hours and the coefficients were obtained by least squares.

Coupling Function	CC	PE
$p^{1/2}v^{4/3}B_T\sin^6\left(\frac{\theta}{2}\right)$	0.89	0.78
vB_s	0.86	0.69
$vB_T\sin^2\left(\frac{\theta}{2}\right)$	0.83	0.56
$vB_T^2\sin^4\left(\frac{\theta}{2}\right)$	0.88	0.75
$p^{1/2}vB_T\sin^4\left(\frac{\theta}{2}\right)$	0.89	0.77

NARX Recurrent Neural Network

NARX recurrent neural networks [Diaconescu, 2008; Pallocchia et al., 2005] can be represented by



The learning algorithm used was back propagation. The number of hidden layers was fixed to 20.

Coupling Function	CC	PE
$p^{1/2}v^{4/3}B_T\sin^6\left(\frac{\theta}{2}\right)$	0.92	0.80
vB_s	0.90	0.78
$vB_T\sin^2\left(\frac{\theta}{2}\right)$	0.88	0.75
$vB_T^2\sin^4\left(\frac{\theta}{2}\right)$	0.87	0.77
$p^{1/2}vB_T\sin^4\left(\frac{\theta}{2}\right)$	0.90	0.77

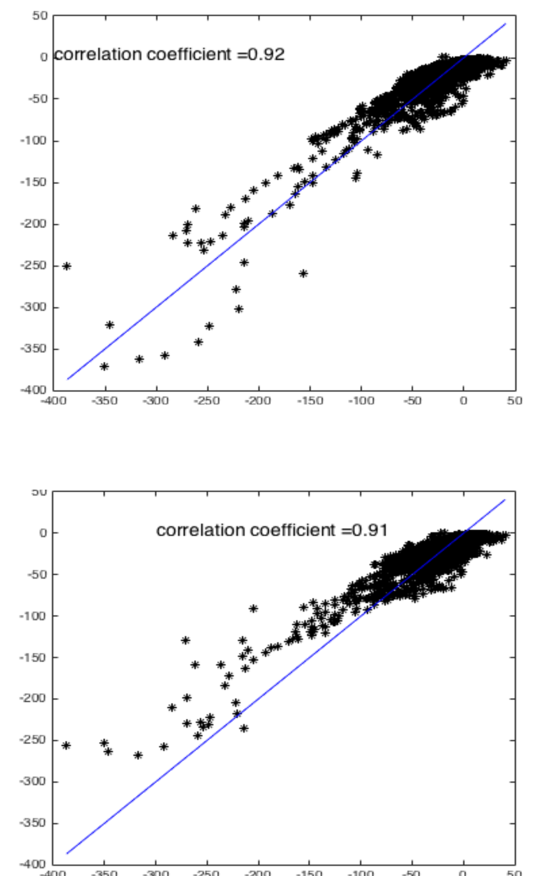
NARMAX

NARXAX models [Billings et al. 1989] can be represented by:

$$\hat{y}(t) = F[\hat{y}(t-1),\ldots,\hat{y}(t-n_y),$$
$$u(t-1),\ldots,u(t-n_u),\ldots,$$
$$e(t-1),\ldots,e(t-n_e)] + e(t)$$

Where F was a nonlinear polynomial set to degree 2, e represents noise terms, and all the maximum lags were set to 2.

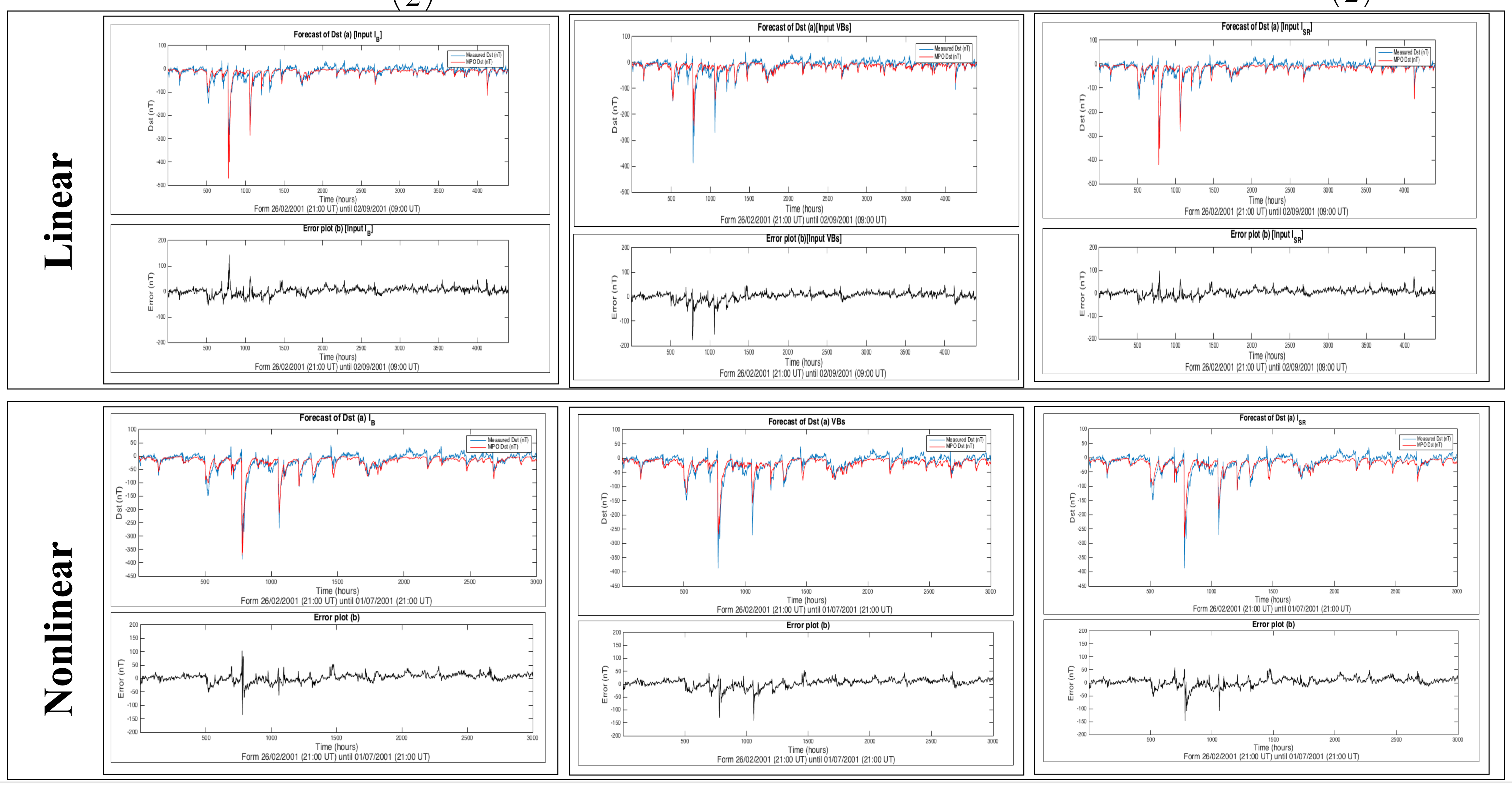
Coupling Function	CC	PE
$p^{1/2}v^{4/3}B_T\sin^6\left(\frac{\theta}{2}\right)$	0.92	0.82
vB_s	0.91	0.78
$vB_T\sin^2\left(\frac{\theta}{2}\right)$	0.88	0.75
$vB_T^2\sin^4\left(\frac{\theta}{2}\right)$	0.87	0.71
$p^{1/2}vB_T\sin^4\left(\frac{\theta}{2}\right)$	0.91	0.77



Scatter plots showing the correlation between the measured Dst index and the model predicted output for the $p^{1/2}v^{4/3}B_T\sin(\theta/2)$ (top) and vB_s (bottom) NARMAX models.

Linear vs. Nonlinear

$p^{1/2}v^{4/3}B_T\sin^6\left(\frac{\theta}{2}\right)$ $\quad vB_s \quad$ $p^{1/2}vB_T\sin^4\left(\frac{\theta}{2}\right)$



Conclusions

The result from all modelling methods generally agree. In all methods the leading models in terms of accuracy were models employing the coupling functions by Boynton et al [2011] as inputs.

The nonlinear methods of NARMAX and NARX recurrent neural networks have a generally higher performance than the linear method.

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