System Identification, Lyapunov Dimension, and NARMAX Forecast of Geomagnetic Indexes

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Prediction of Geospace Radiation Environment and solar wind parameters

PROGRESS, a new Horizon 2020 f
project

Outline

- Introduction
- Problems
- Dynamical-information approach to system identification
- Optimization methods: genetic algorithms
- Estimation of Lyapunov dimension
- Guaranteed prediction
- Conclusion

Objectives

1. An accurate and reliable forecast of space weather hazards.

2. Developing operational tools to predict and forecast them

PAPERS

- Prediction of Geospace Radiation Environment and Solar Wind Parameters, Geophysical Research Abstracts Vol. 17, EGU2015-PREVIEW, 2015 EGU General Assembly 2015.

Problem Description

Forecast of the evolution of geomagnetic indices

Our research concerns improvement and new development of models based on data driven modelling, such as CNN and NARMAX. Existing models for Dst and Kp will be analysed and verified with the aim of finding weaknesses and to suggest improvements. Solar wind and geomagnetic indices shall also be analysed in order to develop models for the identification of features, such as (but not limited to) shocks, sudden commencements, and substorms. Such categorisation will aid the model development and verification, and can also serve as alternative approach to models providing numerical input-output mapping. In addition to the development of Dst and Kp models new models will be developed to forecast AE. The models will be implemented for real-time operation at IRF and data and plots will be provided on a web server.

Introduction

1. This report starts with the physical basis and a brief description of the system approach. Following that, several examples illustrate practical issues in temporal and spatiotemporal prediction and bilinear modeling.

2. An approach based on nonlinear dynamical models and local Lyapunov exponents are used to analyze measurements of the geo-magnetic indexes and solar wind parameters.

Problem Description

- Recursive, robust bilinear dynamical model (RRBDM). It has minimal complexity and the same prediction limit as NARMAX. RRBDM provides forecasts of the Dst and Kp indices based on new robust algorithms and is driven by real time solar wind parameters measured at L1 with a time shift to account for the propagation of the solar wind to the terrestrial magnetopause and the real
- time Dst and Kp indices.

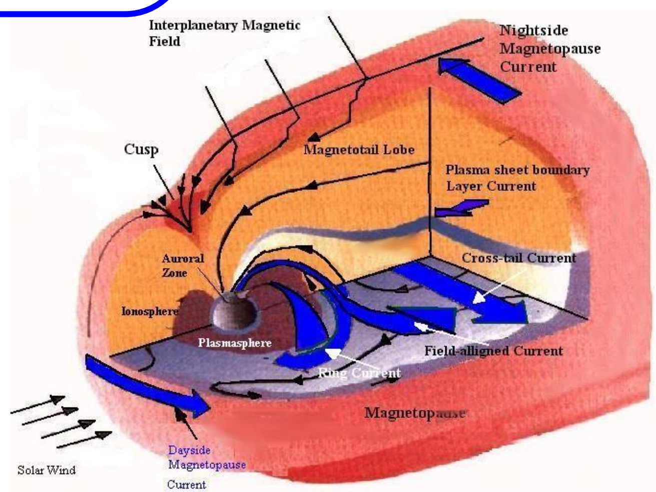
Problem Description

- The second, the Guaranteed NARMAX Model (GNM) also provides predictions of the Dst index. Its main advantage is that it delivers an increased prediction reliability in comparison to earlier SRI models.
- Guaranteed prediction of geomagnetic indexes

Dynamical-information forecasting of geomagnetic indexes

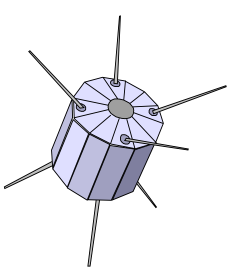
Magnetosphere is considered as a nonlinear complex dynamical system

Kp,AE,Dst indexes



Dst is sought for as an output of a nonlinear dynamical “black-box”

Solar wind parameters



Data are from OMNI2 database:
http://nssdc.gsfc.nasa.gov/omniweb/
and Kyoto WDC for Geomagnetism:
http://swdcdb.kugi.kyoto-u.ac.jp/

Problem (cont'd)

- Predicting the evolution of geomagnetic indexes in solar wind flows.
- Limit of predictability and Lyapunov exponents
- Analyze the variation of Lyapunov exponents of solar and geomagnetic activity indices during coronal mass ejections
- Comertial forecasting services

Dynamical-information approach

- Dynamical-information approach is based on the "black-box" models and ***Lyapunov exponents*** to describe magnetospheric dynamics.
- Reconstruction of the dynamical model is based upon the application of **multiobjective learning algorithms** to identification of model's structure and parameters.
- *A* forecasting algorithm based on ***Lyapunov exponents*** is also proposed.

Lyapunov Spectrum

• The analysis has been carried out using the technique of adaptive LE estimation adopted from previous works;
• It is shown that the LE of these solar and geomagnetic activity indices varies rapidly during CMEs.
• The variation in LEs creates a pattern as a precursor for the forthcoming CME.
• This precursor, which is an oscillation in the values of LEs, begins several steps sooner than the CME's occurrence.
•Then, during the CME, the LEs decrease to a small positive or a negative value, which demonstrates that during an anomaly such as a CME the chaotic characteristics of solar and geomagnetic activity indices decrease and solar and geomagnetic activity indices follow more regular dynamics.

Mathematical Models

$$y(t) = \sum_{i=1}^{\infty} \int_0^{\infty} \dots \int_0^{\infty} h_i(\tau_1, \dots, \tau_i) \prod_{j=1}^{i} u(t - \tau_j) d\tau_j$$

y(k) = F[y(k - 1),…,y(k - n), u(k - 1),
…, u(k - n), ξ(k - 1),…,ξ(k - n)+ξ(k)]

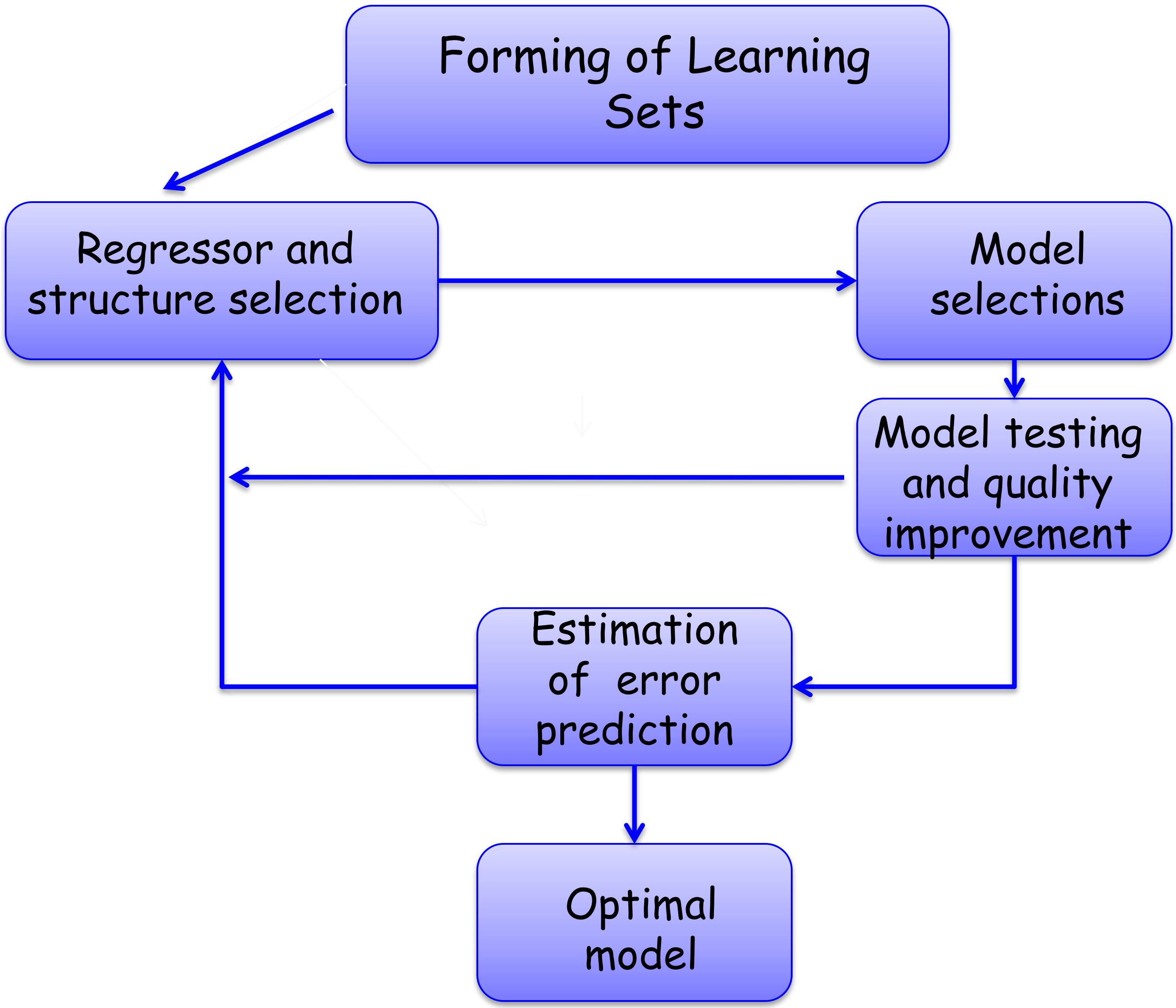
$$J = \sqrt{((\hat{y}(k) - y(k))^2) / (\sum (y(k) - \bar{y}(k))^2)}$$

Bilinear models

Methods of identifying bilinear systems from recorded input-output data have been proposed. The identified bilinear model is then used to forecast the evolution of the Dst index . For the investigation of robust forecasting, we perform a simulation study to demonstrate the applicability and the forecasting performance.

$$\dot{x}(t) = A_0 x + \sum_{i=1}^{m} B_i x(t) u_i(t),$$

$$y(t) = Dx(t),$$


Forming of Learning Sets
Regressor and structure selection
Model selections
Model testing and quality improvement
Estimation of error prediction
Optimal model

Optimization Problem

$$y(k) = \psi(k-1)^T \boldsymbol{\theta} + \xi(k) \qquad (1)$$

$$\min \quad J(\boldsymbol{\theta})$$
$$\text{subject to} \quad \Theta \in D \qquad (2)$$

Optimization Problem

$$(P_M)\left|\begin{array}{l} \text{minimize } v(x) = \prod_{i=1}^{m} f_i(x) \\ \text{subject to} \quad g_j(x) \leq 0, \ \ j = 1, 2, \ldots, p, \end{array}\right.$$

$$f_i : \mathbb{R}^n \to \mathbb{R} \ (i = 1, 2, \ldots, m) :$$

$$g_j : \mathbb{R}^n \to \mathbb{R} \ (j = 1, 2, \ldots, p)$$

$$\Omega := \{x \in \mathbb{R}^n : g_j(x) \leq 0, \ j = 1, 2, \ldots, p\}$$

Numerical Algorithms

For structure and parameter identification we use three numerical methods:

1. Nonlinear parametric model identification using genetic algorithms
2. Nonlinear optimization with constraints

Numerical results

$$
\begin{aligned}
y(k) = {} & a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) + a_4 y(k-4) + a_5 y(k-5) + a_6 u(k-1) + \\
& + a_7 u(k-2) + a_8 u(k-3) + a_9 u(k-4) + a_{10} u(k-5) + a_{11} u(k-6) + \\
& + a_{12}\, u(k-4) u(k-5) + a_{13} y(k-1) u(k-4) + a_{14} u(k-5) u(k-3),
\end{aligned}
$$

$$
\begin{aligned}
y(k) = {} & a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) + a_4 y(k-4) + a_5 y(k-5) + \\
& + a_6 u(k) + a_7 u(k-1) + a_8 u(k-2) + a_9 u(k-3) + a_{10} u(k-4) + \\
& + a_{11} u(k-5) + a_{12} u(k-6) + a_{13} y(k-5) \cdot u(k-5) + a_{14} y(k-3) \cdot u(k-5) + \\
& + a_{15} u^2 (k-6),
\end{aligned}
$$

$$
\begin{aligned}
y(k) = {} & a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) + a_4 y(k-4) + a_5 y(k-5) + a_6 u(k-1) + \\
& + a_7 u(k-2) + a_8 u(k-5) + a_9 y(k-2) u(k-1) + a_{10} u(k-4)\, u(k-6) + \\
& + a_{11} y(k-3)\, u(k-1) + a_{12} u(k-1) u(k-7) + a_{13} y(k-3) u(k-2) + a_{14} u(k-2) u(k-5) + \\
& + a_{15} u(k-7) u(k-12).
\end{aligned}
$$

Numerical results

$y_i = 1.36\, y_{i-1} - 4.99\, u_{i-1} - 0.18\, y_{i-2}\, u_{i-1} - 0.57\, y_{i-4} -$

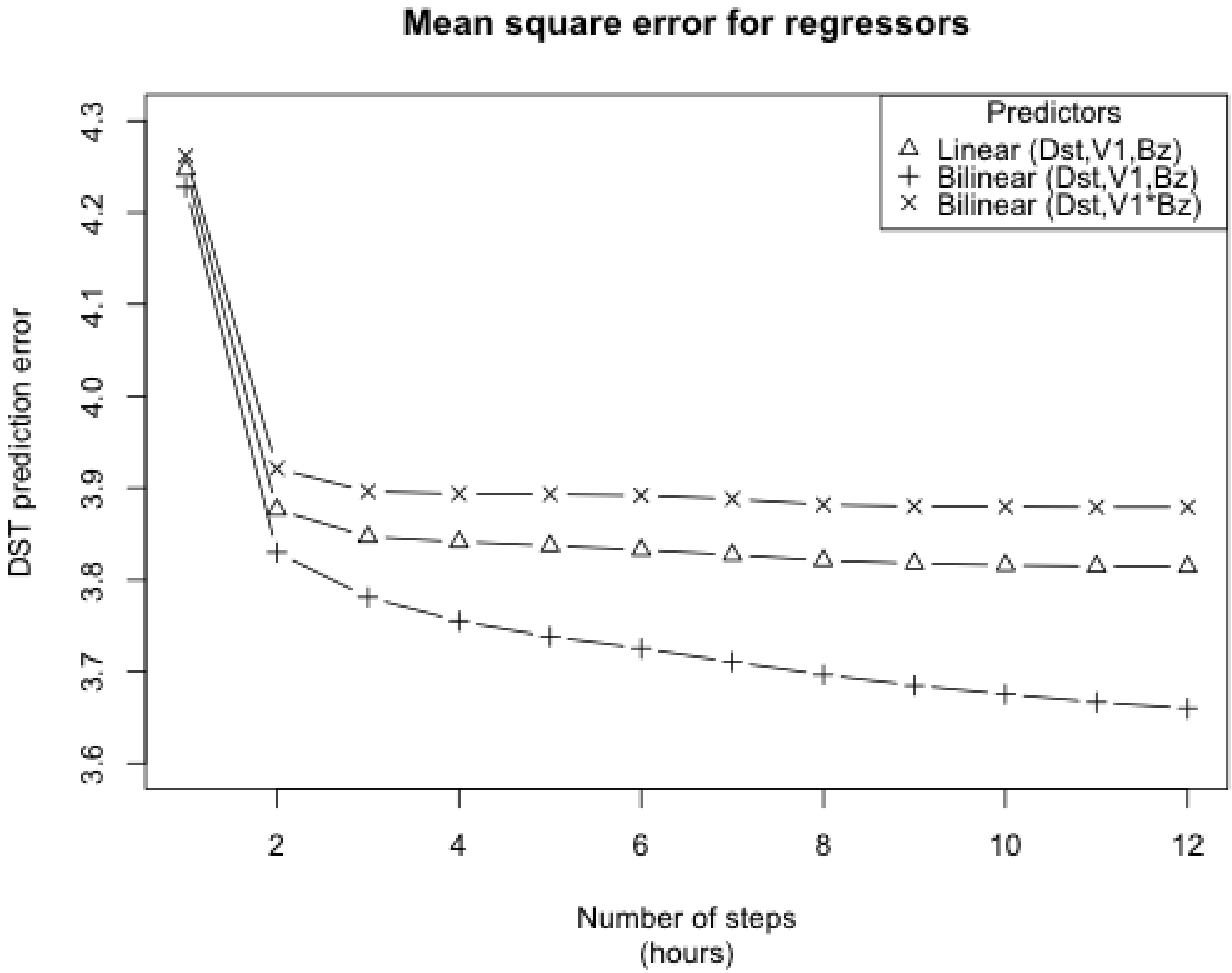
$-1.43\, u_{i-4}\, u_{i-6} - 0.75\, y_{i-2} + 0.53\, y_{i-3} + 0.1\, y_{i-3}\, u_{i-1} +$

$+0.36\, y_{i-5} + 0.92\, u_{i-6}\, u_{i-7} + 2.71\, u_{i-2} + 0.08\, y_{i-3}\, u_{i-2} +$

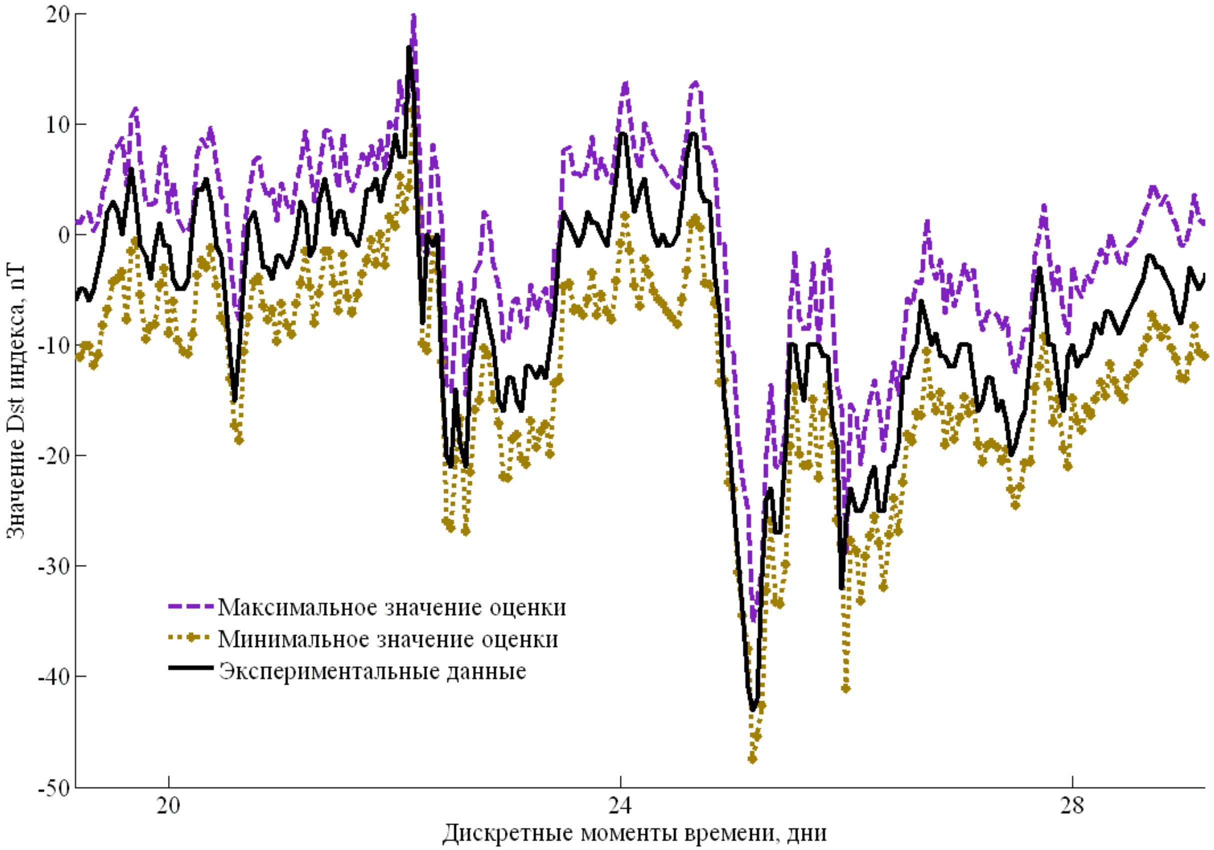
$+0.78\, u_{i-2}\, u_{i-5} - 0.91\, u_{i-5} + 0.25\, u_{i-7}\, u_{i-12}$

$$\Xi = \sqrt{((\hat{y}(k) - y(k))^2)/(\sum(y(k) - \bar{y}(k))^2)}$$

Numerical results



Results (Guaranteed prediction)



Risk analysis

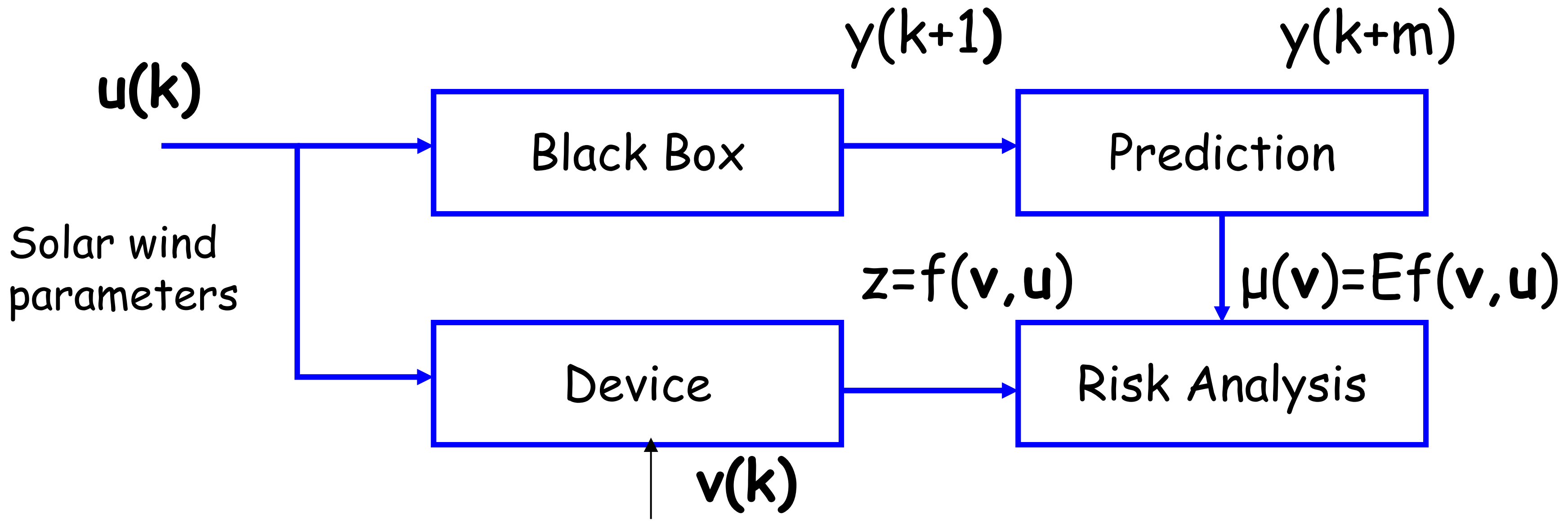


Fig. 1 Prediction and Risk Analysis

Optimization problem with constraints on risk

Let z=f(v,u) be a loss function of a device depending upon the control vector v and a random vector u. The control vector v belongs to a feasible set V, satisfying imposed requirements. We assume that the random vector u has a probability density p(u). We can define a function

$$\Phi_\beta(v,\beta) = (\alpha - \beta)^{-1} \int_{f(v,u)>\alpha} (f(v,u)) - \alpha)p(u)du.$$

Optimization model

$$\min \mu(v)$$

$$v \in V, \Phi_\beta(x) \leq C_\beta, \Phi_\gamma(x) \leq C_\gamma.$$

Applications

- Hybrid energy storage device based on supercapacitors
- Space accelerometers
- Superconducting gravimeter
- Lasers

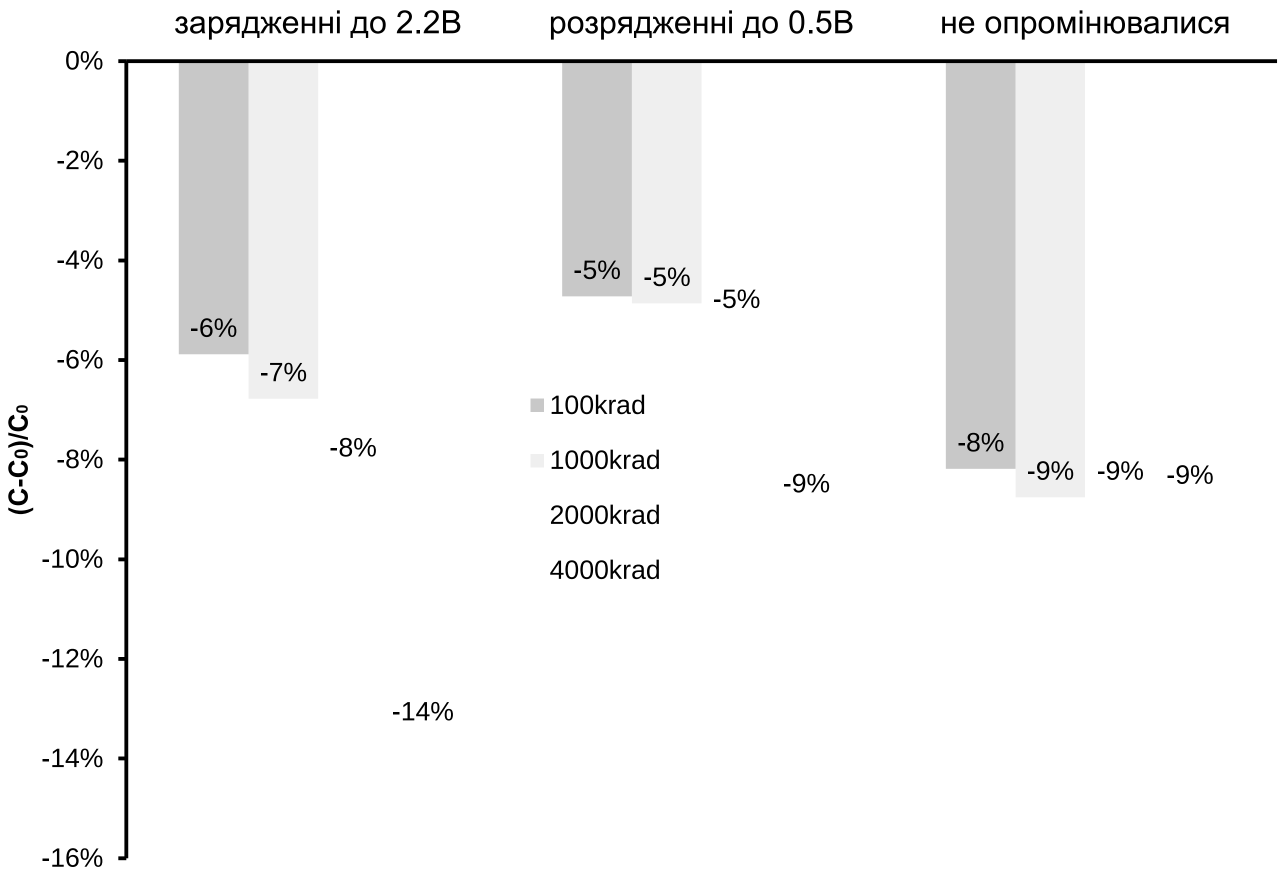
Applications

Hybrid energy storage system based on supercapacitors

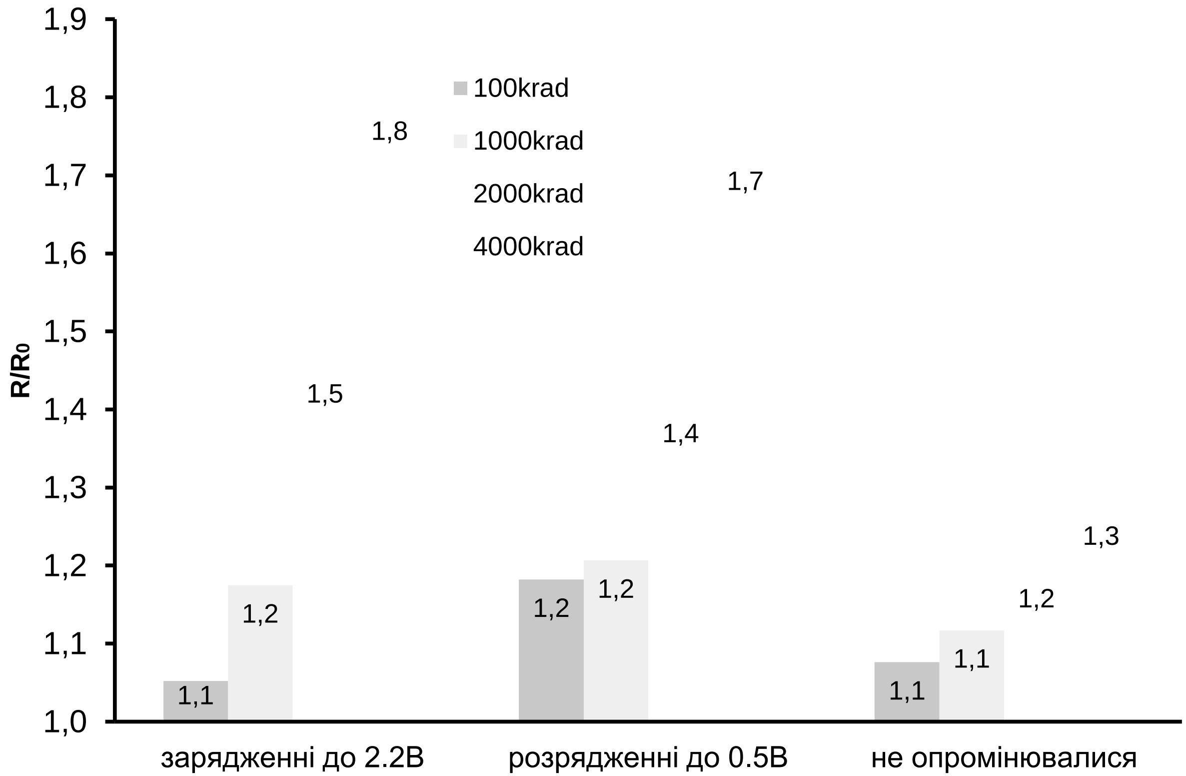
P, кВт

E, Вт·год

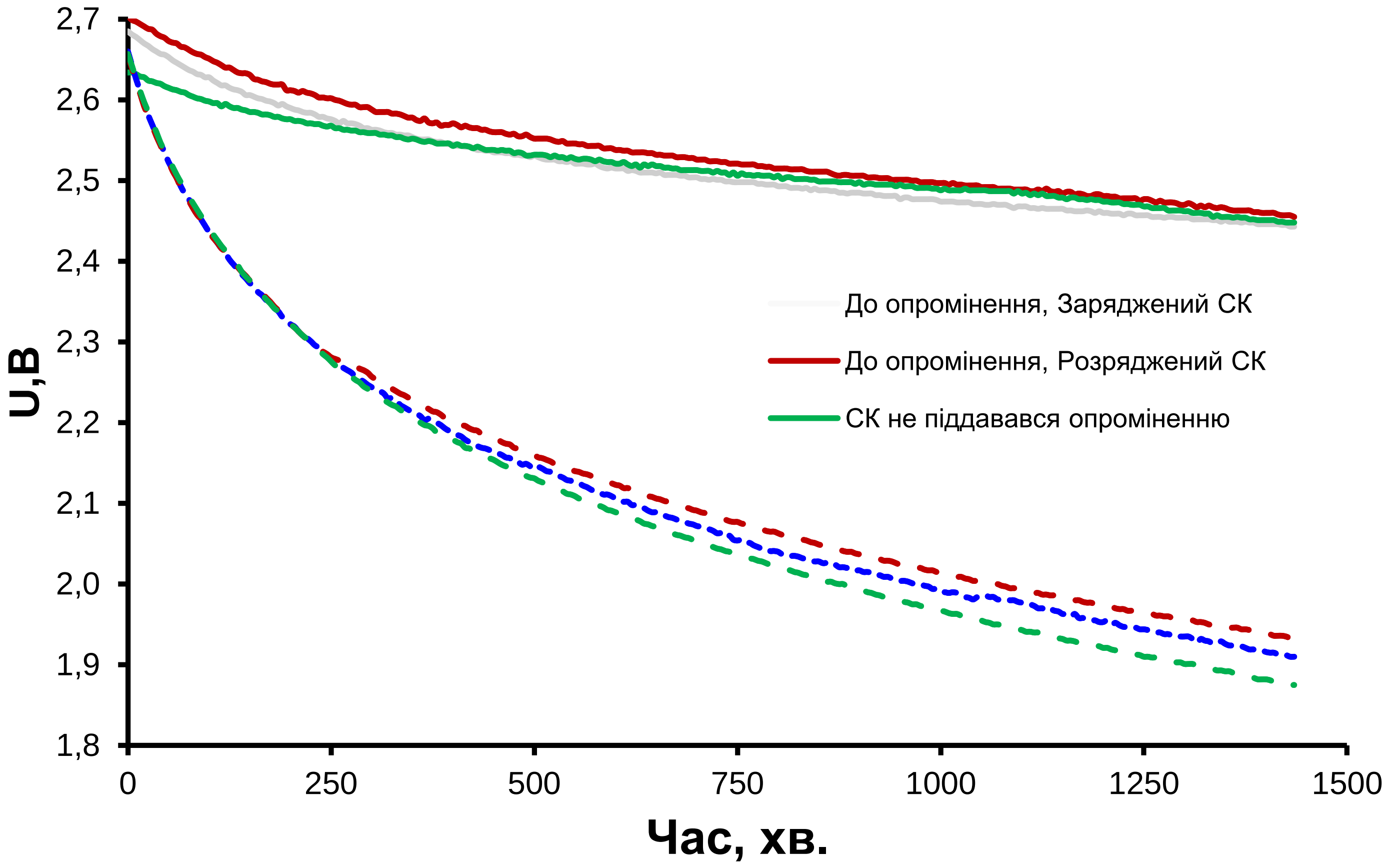
Impact γ-irradiation on capacity of hybrid energy storage device



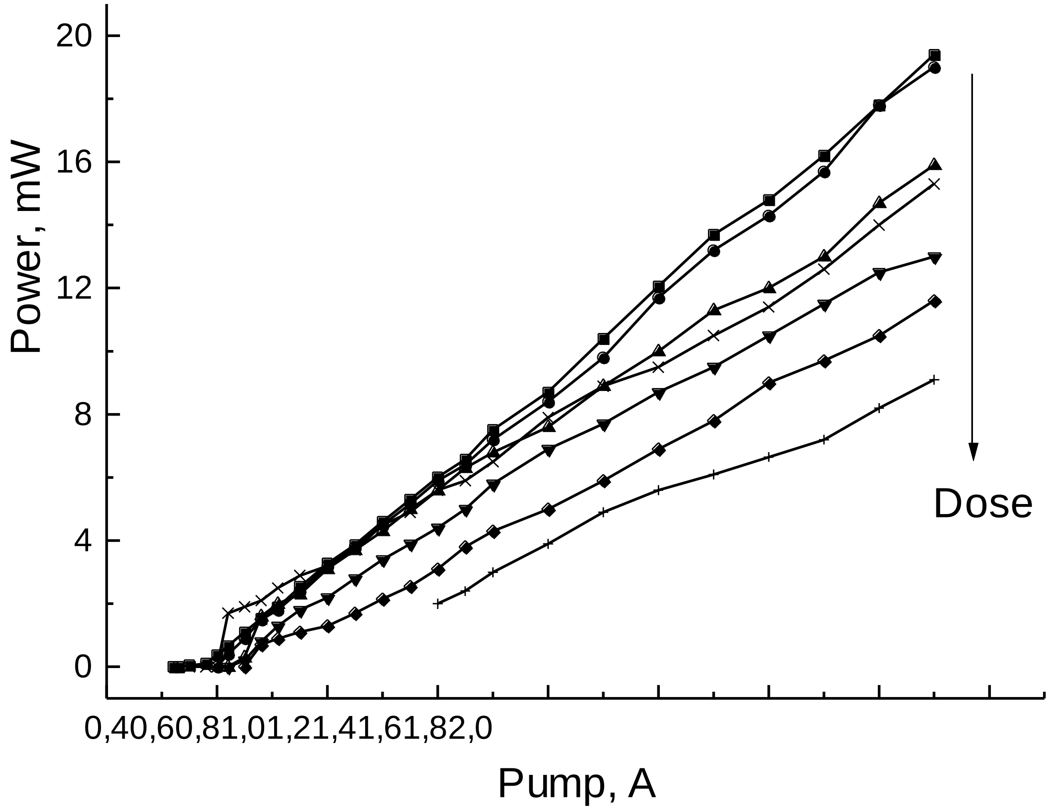
Resistance increase by γ-irradiation of hibrid energy storage device



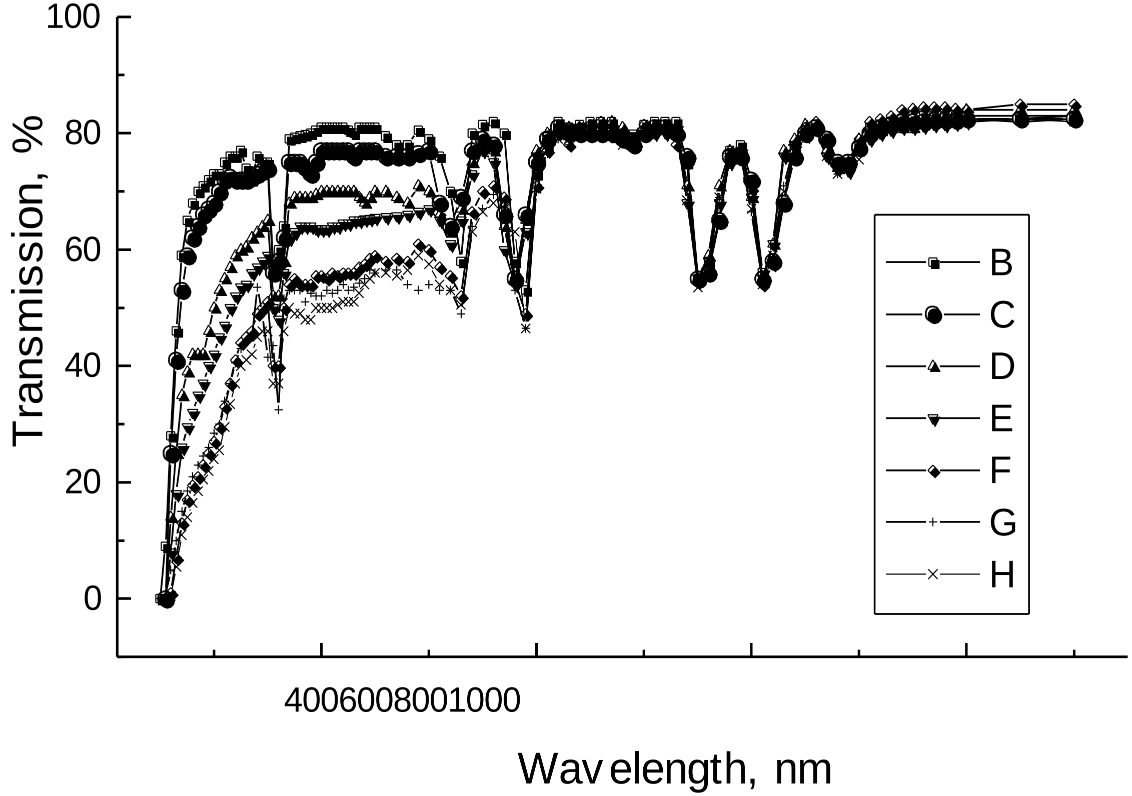
Voltage decreases of supercapacitors before and after γ-irradiation



Output of the diode laser after irradiation by gamma radiation



Transmission of Nd YAG crystal plate



Conclusions

- The following models have been proposed:
  (a) solar wind influences on devices;
  (b) forecasting of ionizing radiation;
  (c) risk assessment in safety analysis.
- We propose a new approach for radiation damage risk assessment of laser elements by cosmic radiation. This approach based on the Conditional Value-at-Risk measure, the expected loss exceeding Value-at-Risk.
- An application of the multicriterion optimization method to the prediction of the geomagnetic indexes. Novel algorithms to the identification of discrete input-output models have been developed.

Conclusions (cont'd)

- Analysis depicts the variation of LEs of these solar and geomagnetic activity indices during these CMEs. The LEs of these solar activity indices begin to vary in such a way that an obvious pattern can be detected about 10 steps sooner before storm begins. In addition, there is a drop off in the largest Lyapunov exponent during magnetic storms and CMEs. This phenomenon depicts that the chaotic characteristics of Dst index decrease during an anomaly such as a CME which has been reported in other natural systems such as occurrence of an epileptic seizure.

- Therefore, it is meaningful to use the variation o f LEs especially for Dst index for al arming systems against space weather hazards. This can be done by using some spectral analysis tools and frequency transforms such as Singular Spectrum Analysis (SSA), Fast Fourier Transform (FFT), and Wavelet Transform as a time-frequency approach. It follows that it is possible to detect a CME by detecting such precursors in frequency or time-frequency domains.

Conclusions (cont'd)

- The simulation results show that the proposed technique provides an efficient method to get the optimum difference equation of the Dst index.
- A numerical bilinear model of Dst-index was built, which characterizes the sporadic change of the magnetic field. It gives reasonable forecast forward for 5-6 hours and can be used to predict the geomagnetic storms.
- An approach to predicting the behavior of Dst-index with the help of local Lyapunov exponents was proposed.
- Procedure implemented for calculating the forecast horizon behavior Dst-index with the use of local Lyapunov exponents.

Conclusions (cont'd)

- In our research we tried to elicit the variation of chaotic trends of geomagnetic activity indices such as Dst indices during CMEs by estimating the Lyapunov exponents via an improved adaptive estimation method based on stochastic optimization.
- Two well-known CMEs are considered in this paper. The first one is the CME which caused the major magnetic storm and the nine hours black out in Quebec, Canada on 13 March 1989 and the second one is the CME on 11 January 1997 which caused the most well-known satellite failure (failure of Telstar 401 satellite).

Conclusions (cont'd)

- Analysis depicts the variation of LEs of these solar and geomagnetic activity indices during these CMEs.
- The LEs of these solar activity indices begin to vary in such a way that an obvious pattern can be detected about 12 steps sooner before storm begins. In addition, there is a drop off in the largest Lyapunov exponent during magnetic storms and CMEs.
- This phenomenon depicts that the chaotic characteristics of Dst index decrease during an anomaly such as a CME .
- It follows that it is possible to detect a CME by detecting such precursors in frequency or time-frequency domains.

Conclusions (cont'd)

- The second, based on physical principles, provides good coverage throughout the whole inner magnetosphere but with significantly lower accuracy.
- The combination of both approaches, as used in the SNB3GEO electron flux model (which combines the data driven NARMAX and physical VERB models), can overcome many of the short comings of the two individual models, generating improved short term forecasts for the whose RB region.
- Long term RB forecasts require the estimation of solar wind parameters at L1 based on remote solar observations. principles based methodologies.

Conclusions (cont'd)

- Energetic electrons within the inner magnetosphere can cause both deep and surface charging of spacecraft operating at GEO and MEO orbits. Reliable forecast of the fluences of these electrons can assist in the mitigation of undesirable effects on spacecraft. Previous forecasts of these fluences exploited either system science or first
- The first, system science approach provides accurate forecasts of electron fluxes but is limited to regions in which continuous data are available, i.e. GEO.

Conclusions (cont'd)

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- Long term RB forecasts require the estimation of solar wind parameters at L1 based on remote solar observations. principles based methodologies.

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The End