ІКД

Comparison NARMAX, Artifical Neural Networks, and localized Lyapunov exponents for geomagnetic indices prediction

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Space Research Institute of NASU & SSAU

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Установа: Інститут космічних досліджень НАНУ-ДКАУ

Доповідач: Іванов Сергій

Information principle of dynamics

The functional has the form:

$$\Phi_p = -\sum_i p_i(\bullet,t)\ln p_i(\bullet,t) + \beta\sum_i p_i(\bullet,t) l_i t + \mu\sum_i p_i(\bullet,t)\ln\|\varepsilon(0)\| + \gamma\sum_i p_i(\bullet,t) \rightarrow \max$$

Shannon's entropy

Euler-Lagrange multiplexes

Con Divergence rate

Initial conditions

Rationing

Optimal distribution:

$$p_i(\bullet,t) = \frac{\|\varepsilon_i(0)\|^{\mu}\exp(\beta l_i t)}{\sum_i \|\varepsilon_i(0)\|^{\mu}\exp(\beta l_i t)},$$

denoting the norm as d :

$$d_i^t = \left(d_i^0\right)^{\mu}\exp\left(\beta l_i t\right).$$

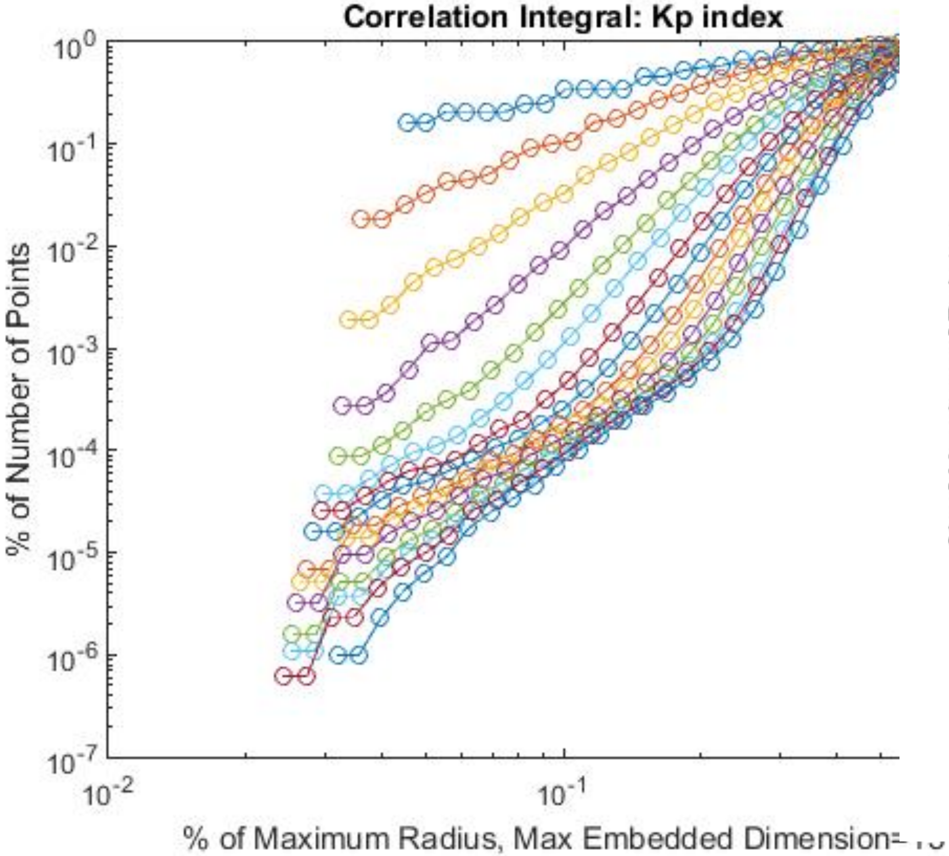
Lyapunov spectrum decomposition

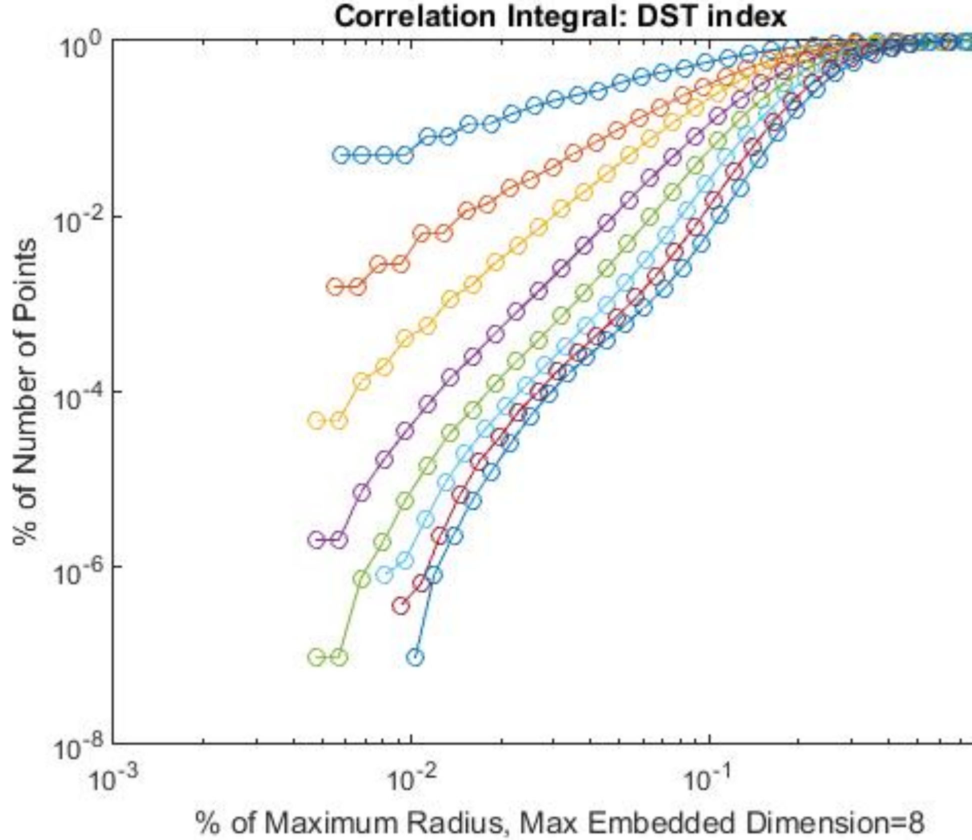
In the application of this principle there is a decomposition which has the following form for the global Lyapunov exponents:

$$\lambda_i = \lim_{t \to \infty} \frac{1}{t} \ln \frac{p_i(\bullet, t)}{p_i(\bullet, 0)} + l_D,$$

$$l_i = \lim_{t \to \infty} \frac{1}{t} \ln \frac{p_i(\bullet, t)}{p_i(\bullet, 0)}, i = \overline{1 \ldots d}.$$

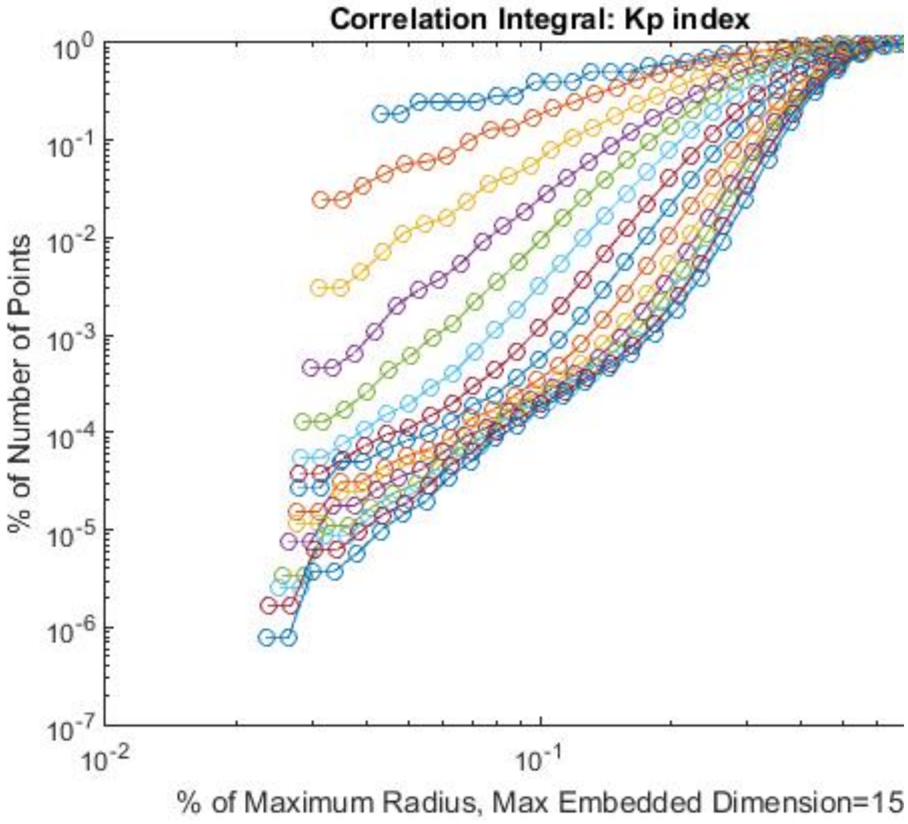
Correlation integrals of Kp & DST indices

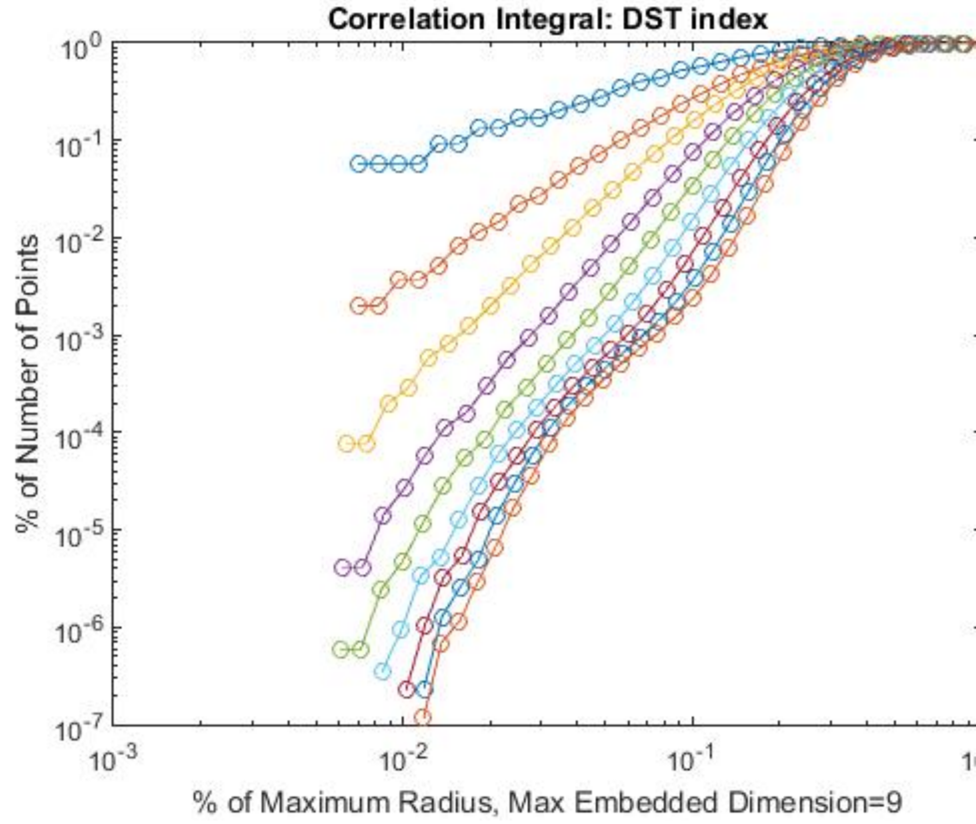




Data for: 06.2015 -04.2016.
Kp(avg) ~ 20;
DST(avg) ~ -18;

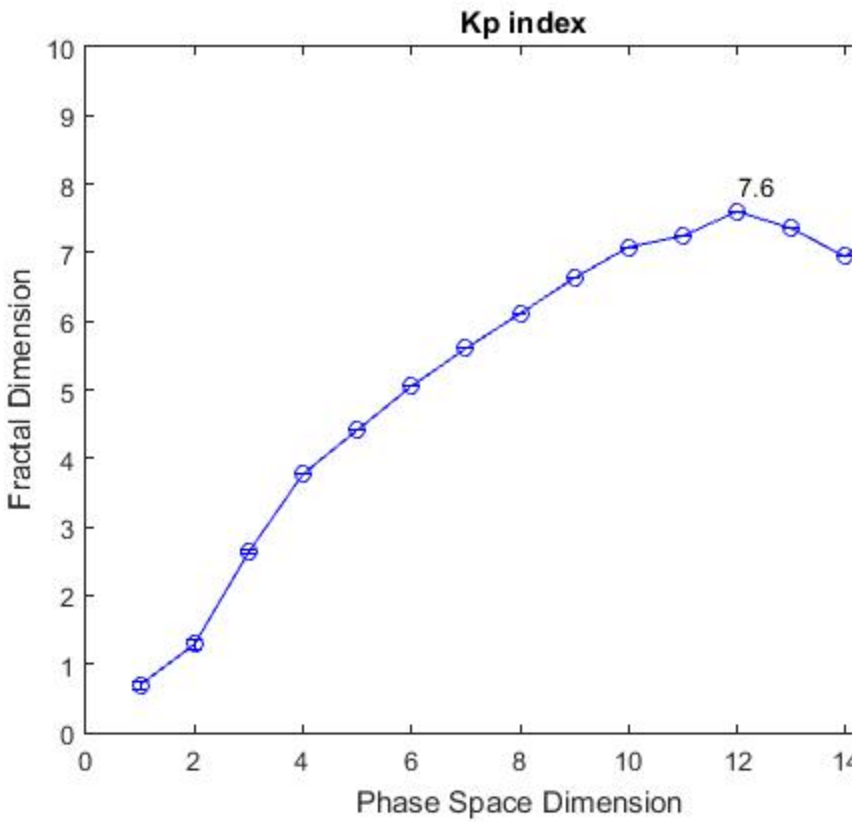
Correlation integrals of Kp & DST indices

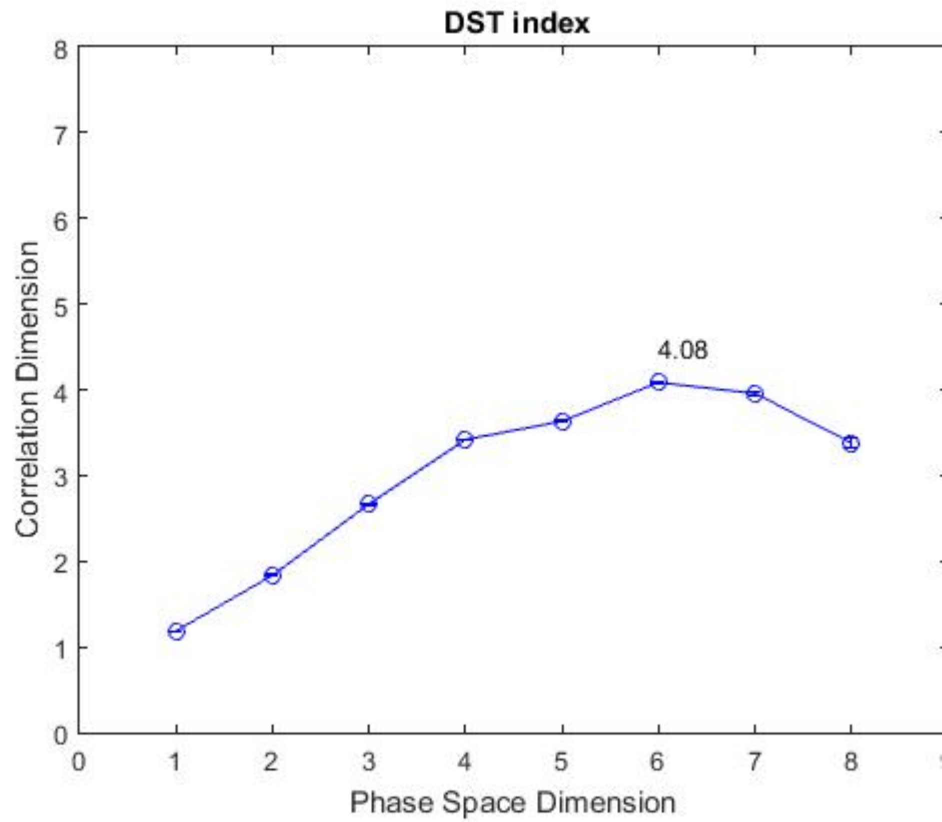




Data for: 03.2013 -12.2013.
Kp(avg) ~ 16;
DST(avg) ~ -9,6;

Correlation and Phase dimensions of Kp & DST indices





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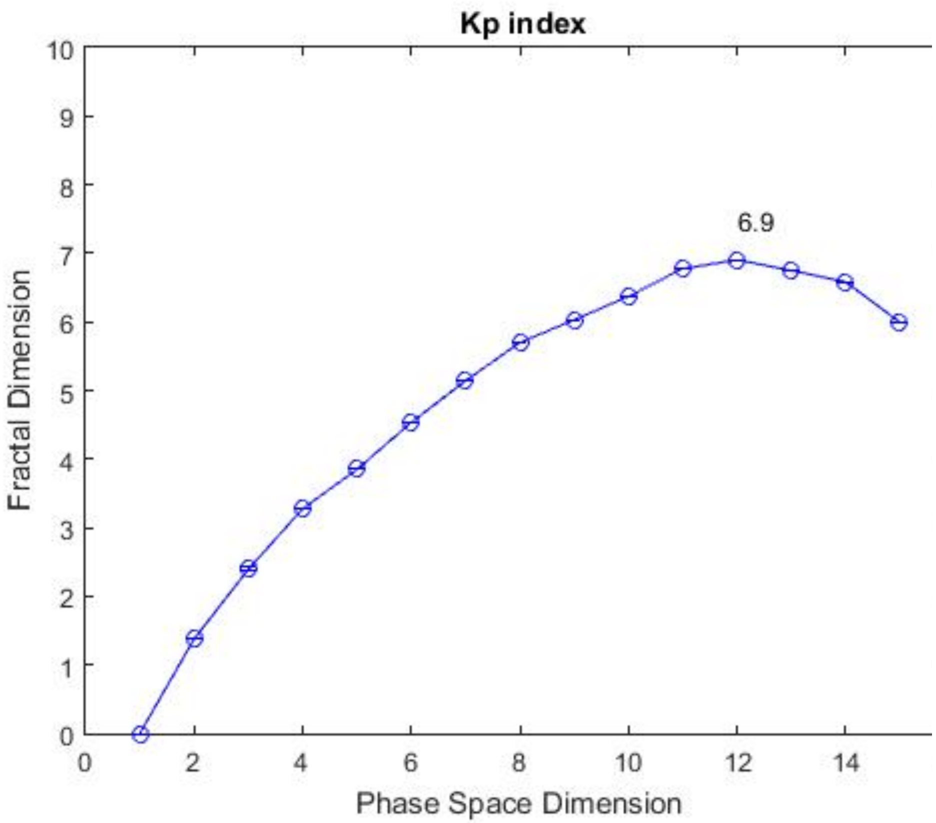

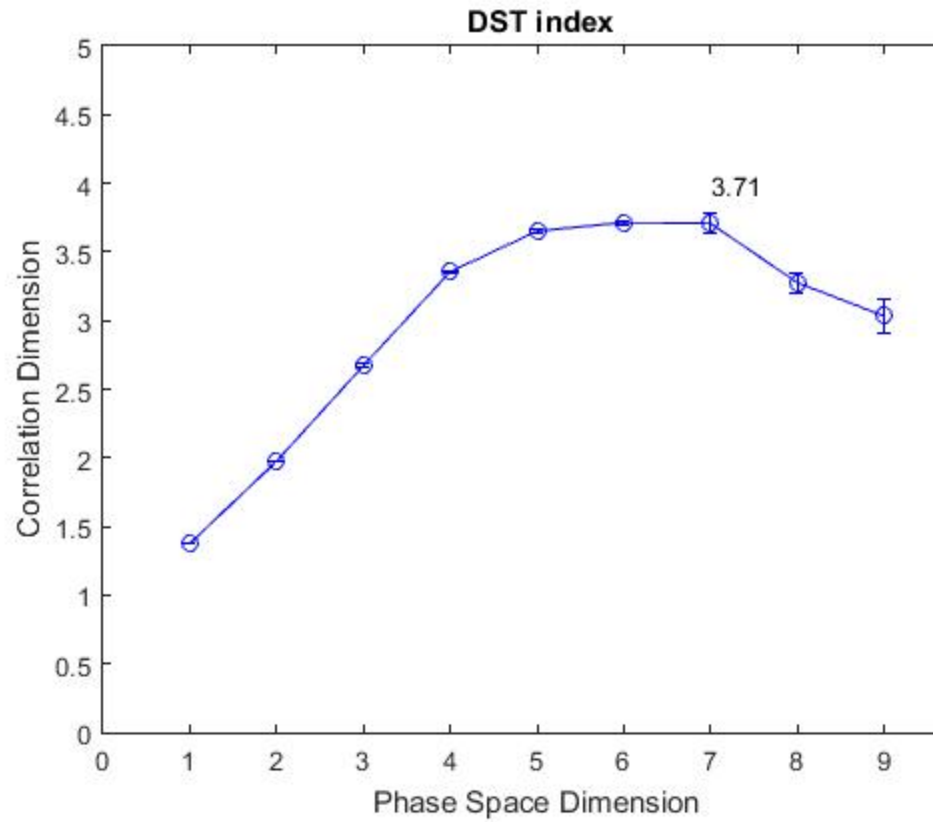

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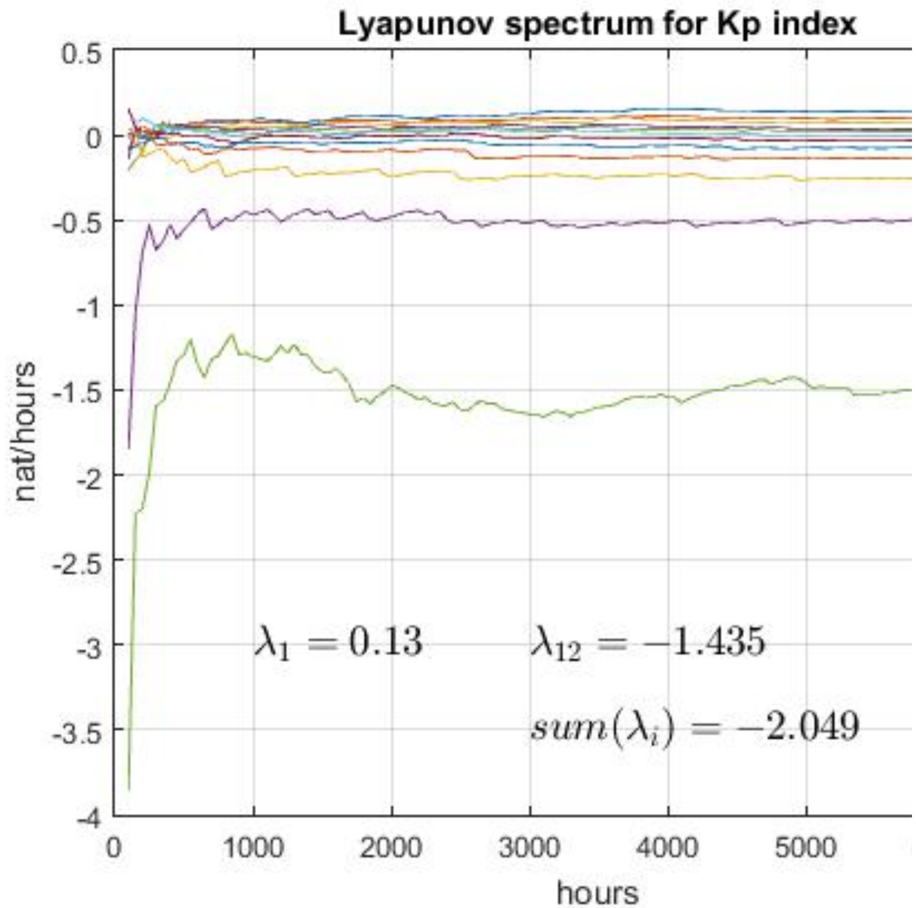
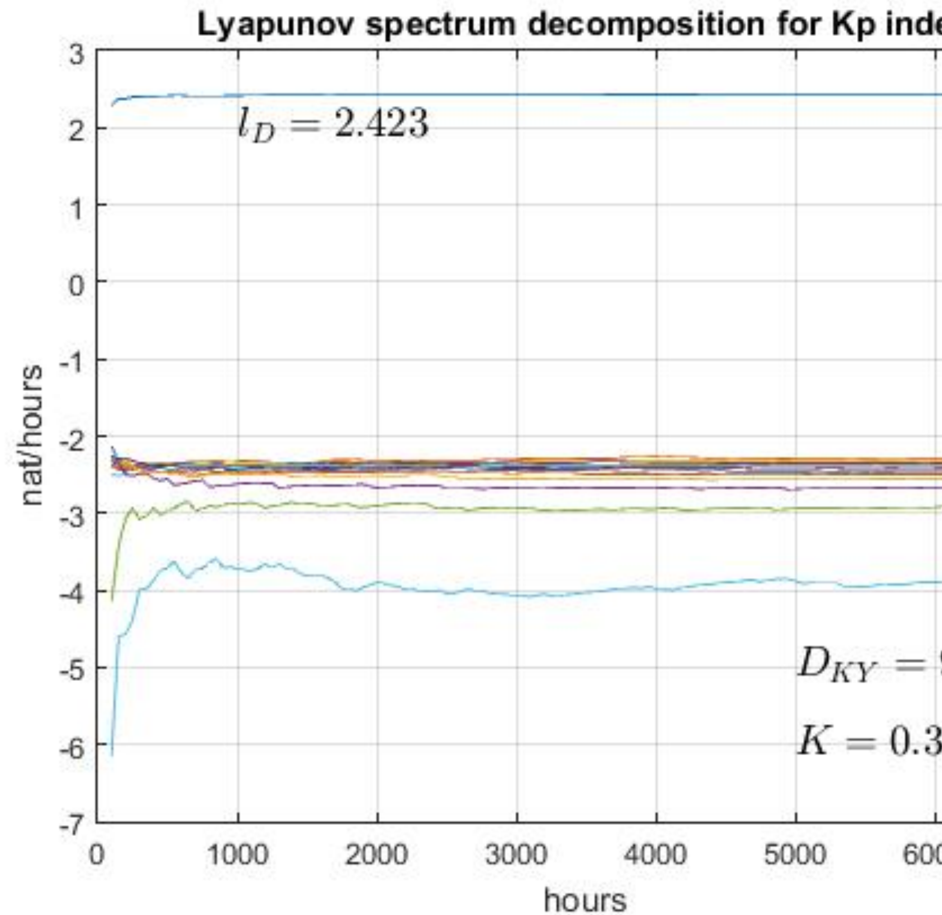
Lyapunov spectrum decomposition





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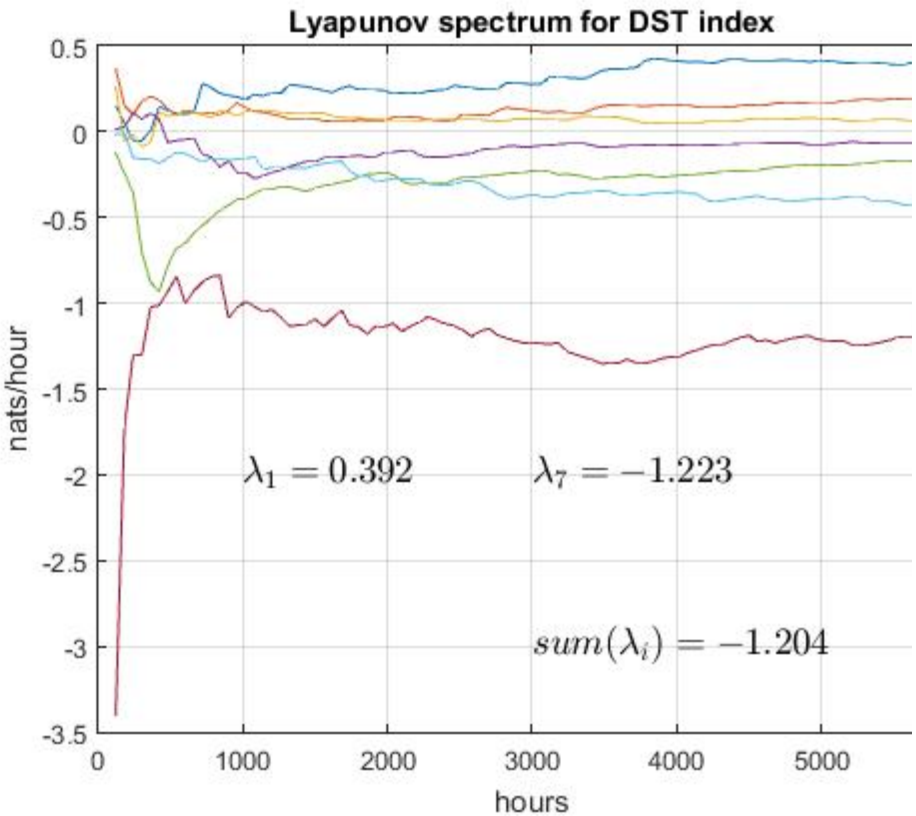
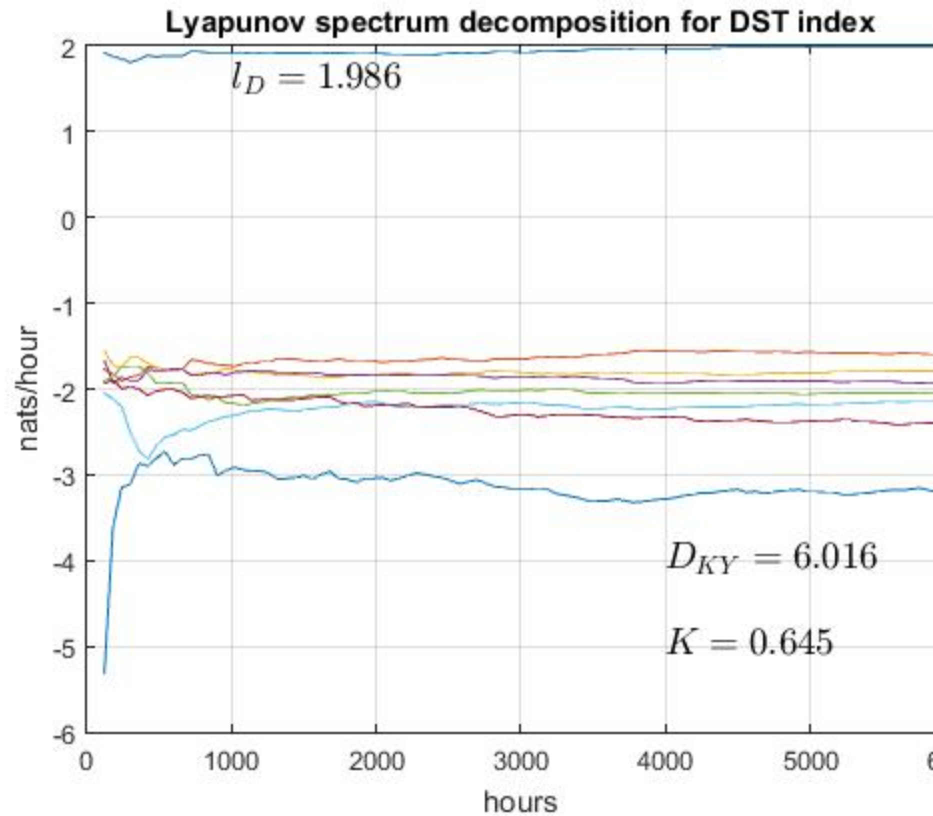
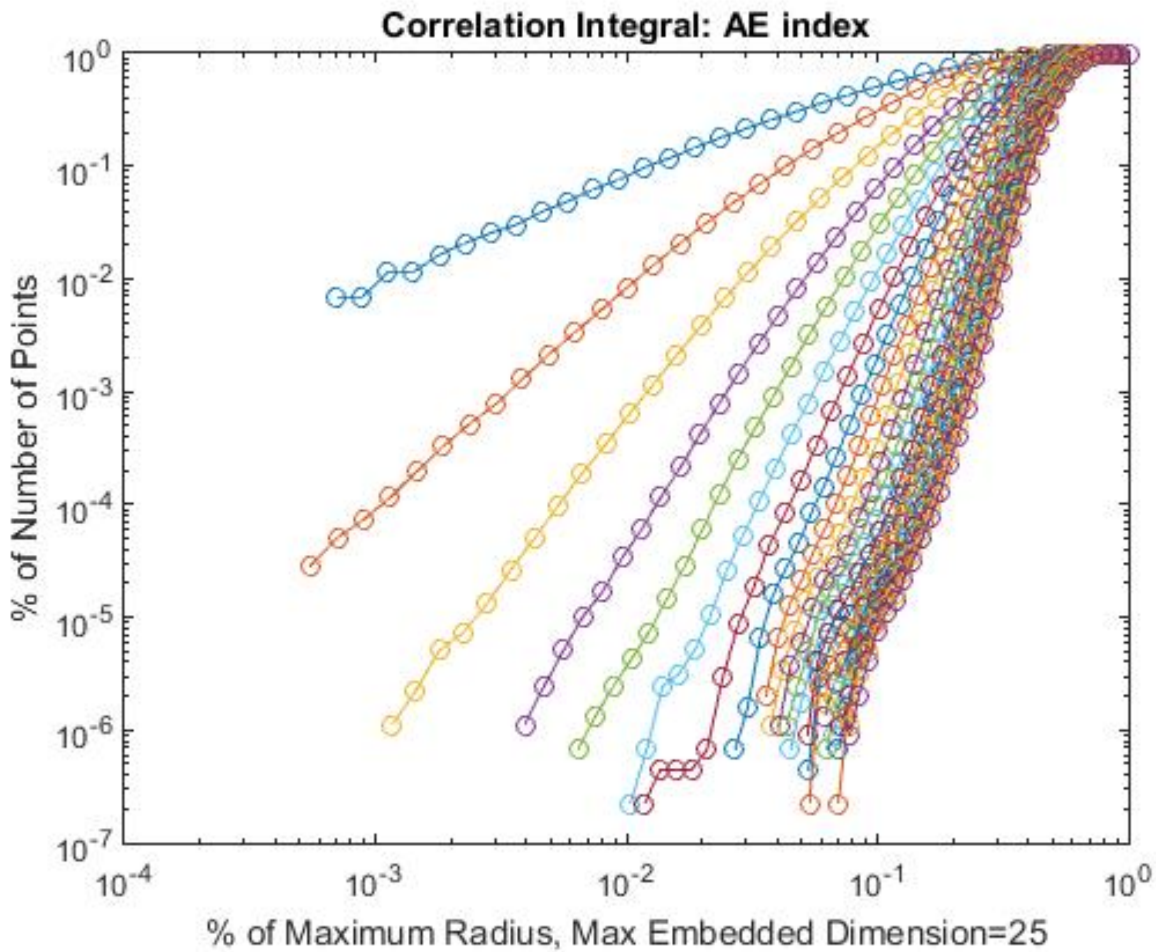




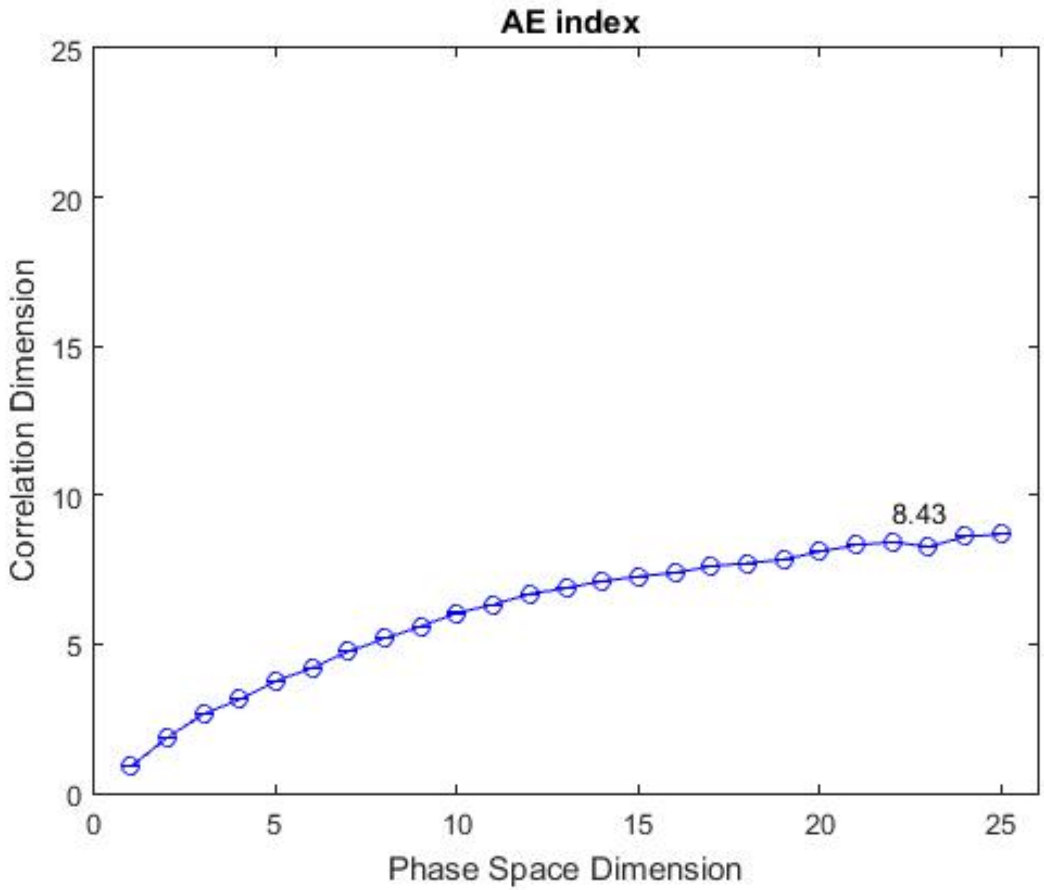
Data for: 06.2015 -04.2016.
Kp(avg) ~ 20;
DST(avg) ~ -18;

Correlation integral of AE index



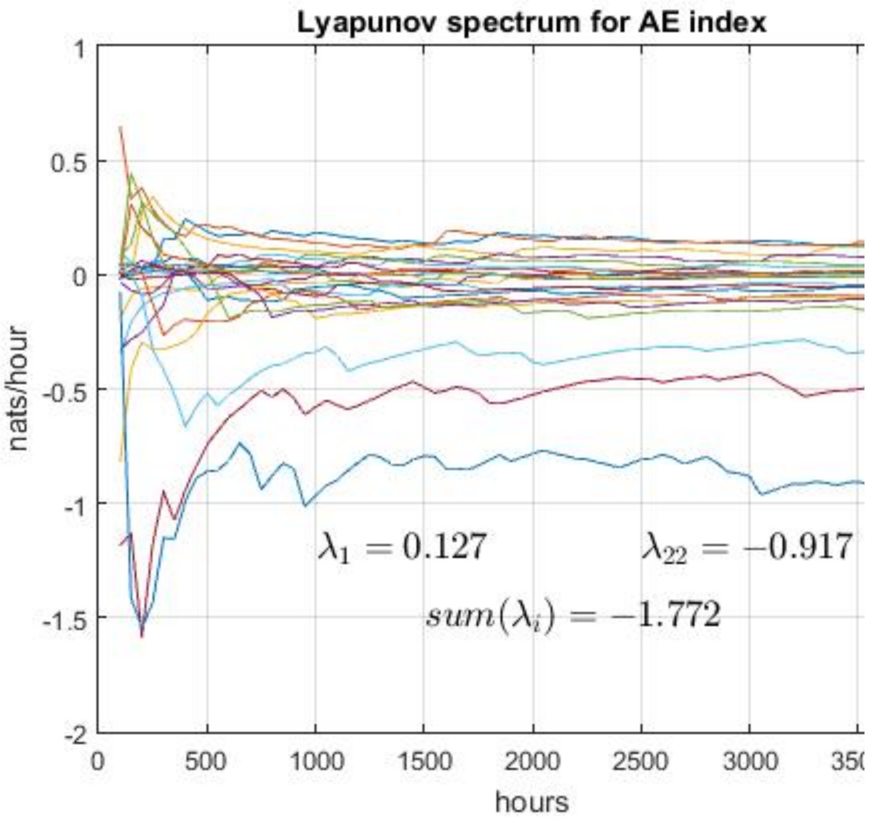
Data for: 06.2015 -04.2016.
AE(avg) ~ 258 nT;

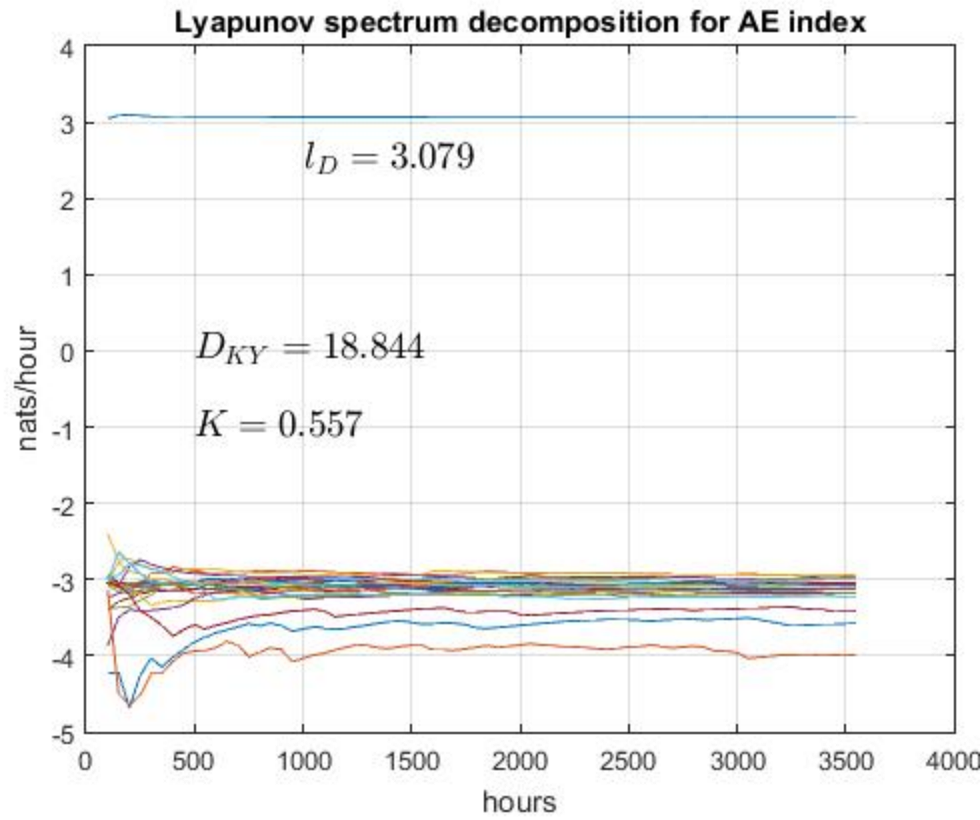
Correlation and Phase dimensions of AE index



Data for: 06.2015 -04.2016.
AE(avg) ~ 258 nT;

Lyapunov spectrum decomposition for AE index





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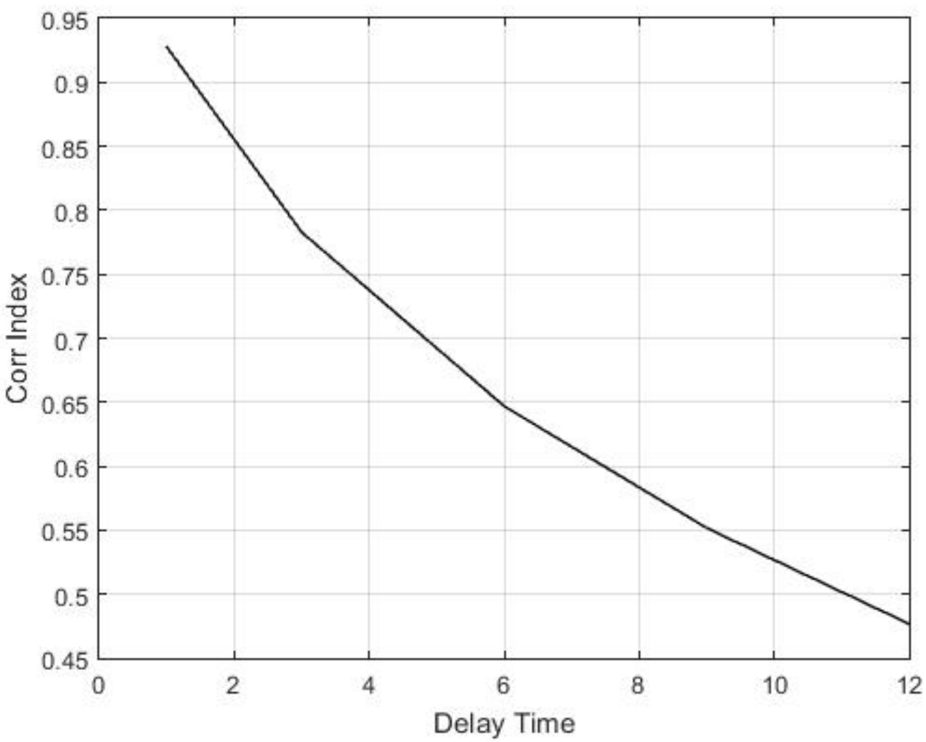
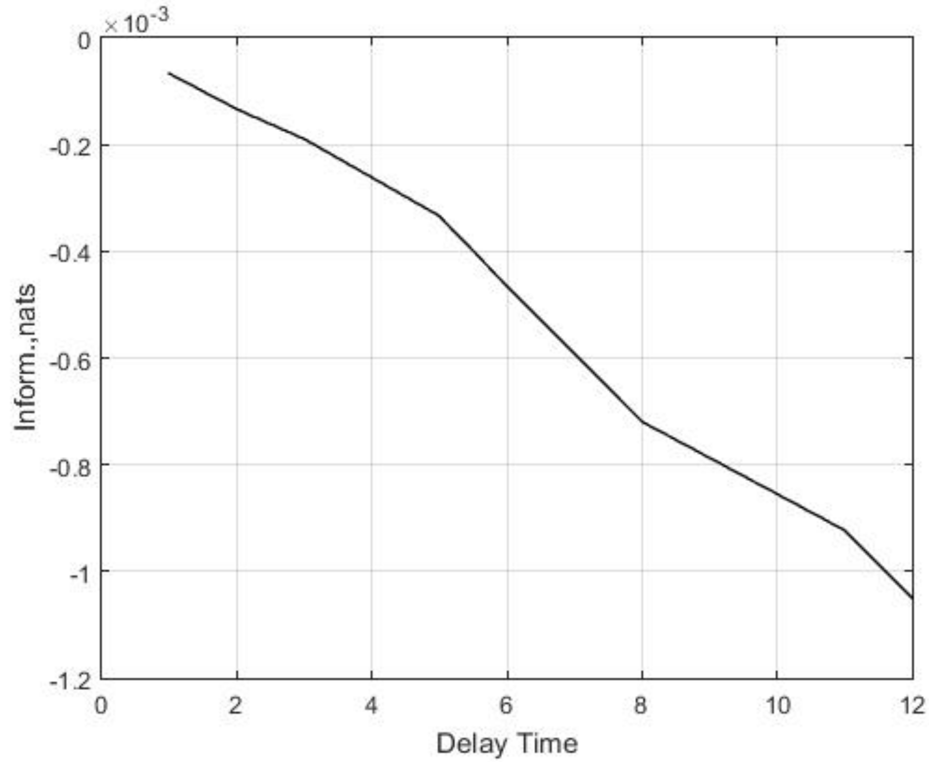
Properties

1. If $l_D > 0$ then $\forall i(dalay\_time):(H_1 - H_i) < 0$





The functional of estimation

The functional with the Euclidean norm:

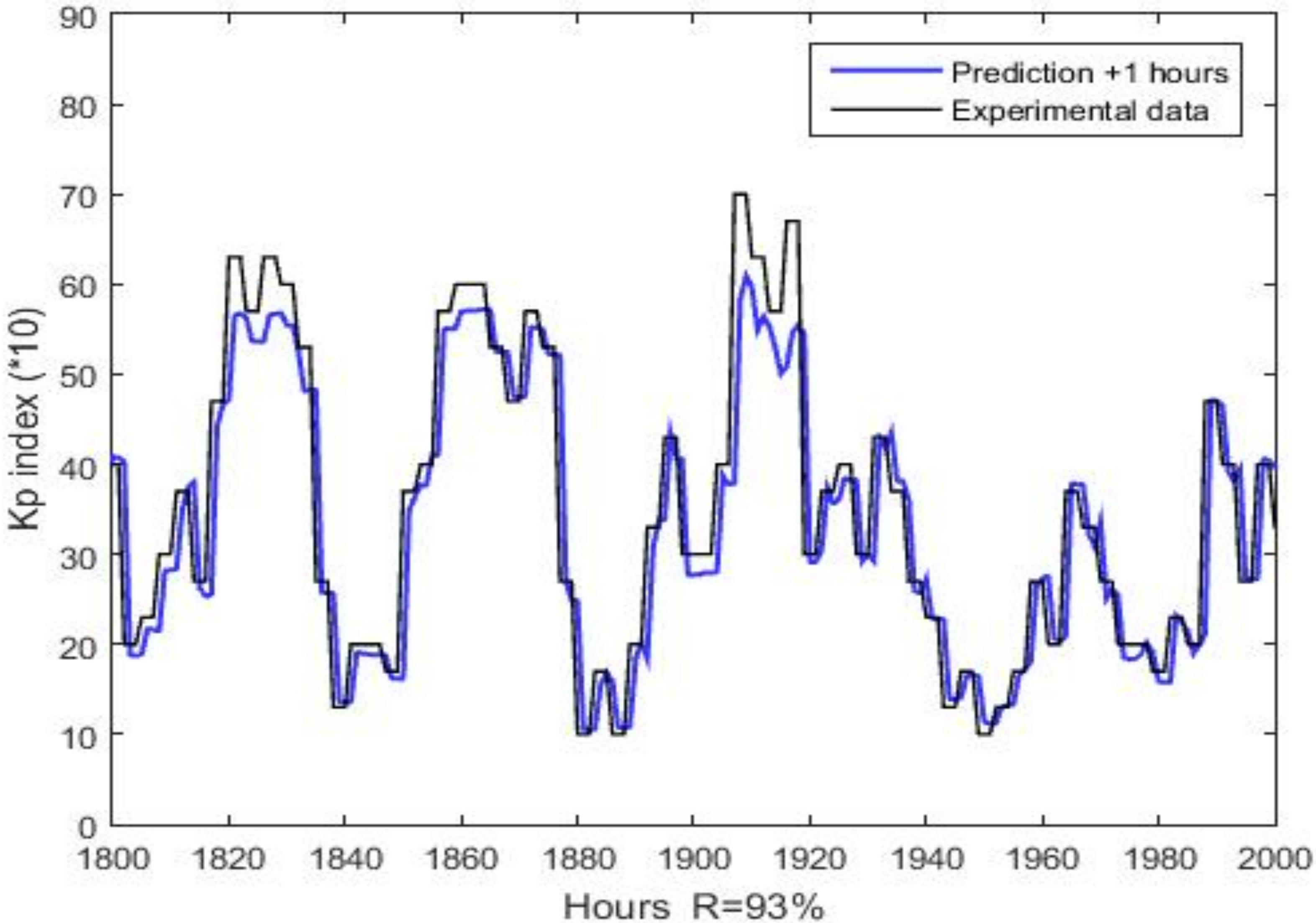
$$d_i^j = \|w\| = \left(w_1^2 + w_2^2 + \ldots + w_m^2\right)^{1/2}$$ is given by:

$$H = -\sum_{i=1}^{m}\sum_{j=1}^{n} p_i^j \ln p_i^j \to \max,$$

$$p_i^j = \frac{1}{a\sqrt{2\pi}}\left[\exp\left(-\frac{\left(d_i^j - \mu\right)^2}{2a^2}\right) + \exp\left(-\frac{\left(d_i^j + \mu\right)^2}{2a^2}\right)\right],$$

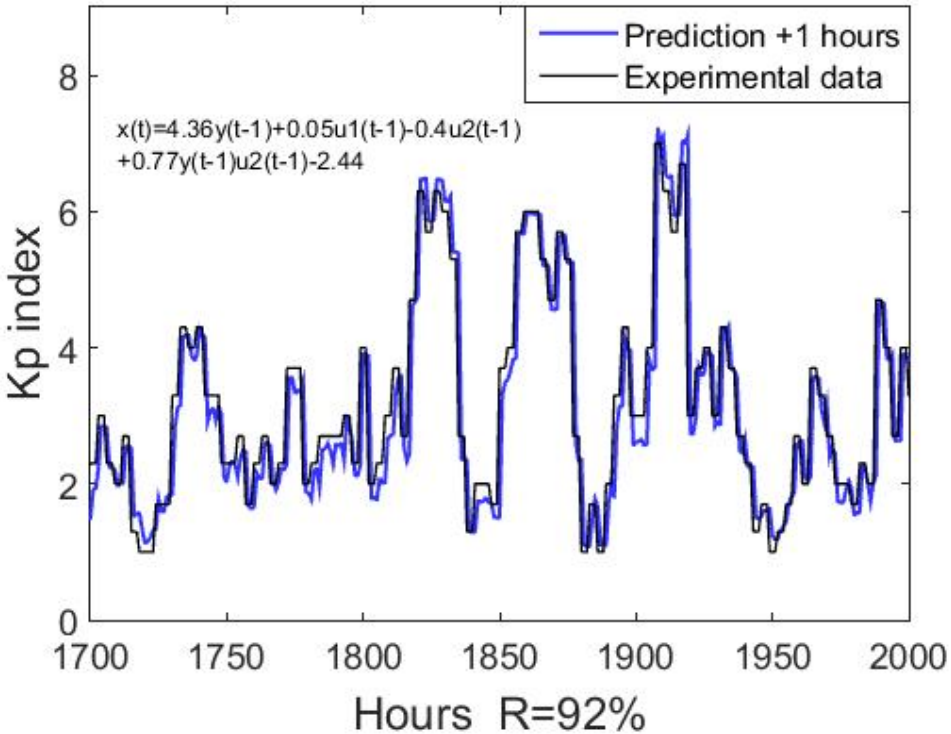
where a – parameter scale distribution ($a > 0$), m – dimension of phase space, n – cardinal number of time series, $f(d_i^j)$ – density distribution of distances (norms) d_i^j, H – Shannon's entropy, μ – position parameter.

NARMAX prediction for Kp index



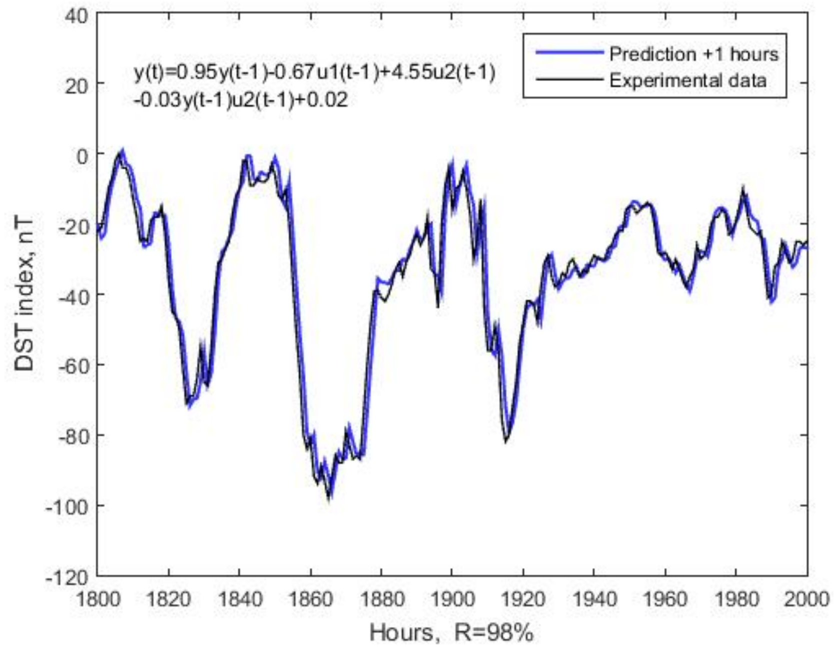
- By using the correlation matrix method for DST and AE index we found the optimal neural network and the optimal NARMAX model are the same. We used one neuron to predict the Kp index. The bilinear summation function of the neuron is shown in Fig. 1. The neuron is presented with a sigmoid activation function. The correlation coefficient is 92% for such an optimal neuron and the optimal NARMAX model without multicolinear members.

- By using the correlation matrix method for DST and AE index we found the optimal neural network and the optimal NARMAX model are the same. We used one neuron to predict the DST and AE indices. The summation function of the neuron is shown in Fig. 2 and Fig. 3. These neurons are presented with a activation function $y(t) = x(t)$. The correlation coefficients are shown in Fig-s.



Prediction +1 hours
Experimental data
x(t)=4.36y(t-1)+0.05u1(t-1)-0.4u2(t-1)
+0.77y(t-1)u2(t-1)-2.44
Kp index
Hours R=92%

Fig. 1



Prediction +1 hours
Experimental data
y(t)=0.95y(t-1)-0.67u1(t-1)+4.55u2(t-1)
-0.03y(t-1)u2(t-1)+0.02
DST index, nT
Hours, R=98%

Fig. 2

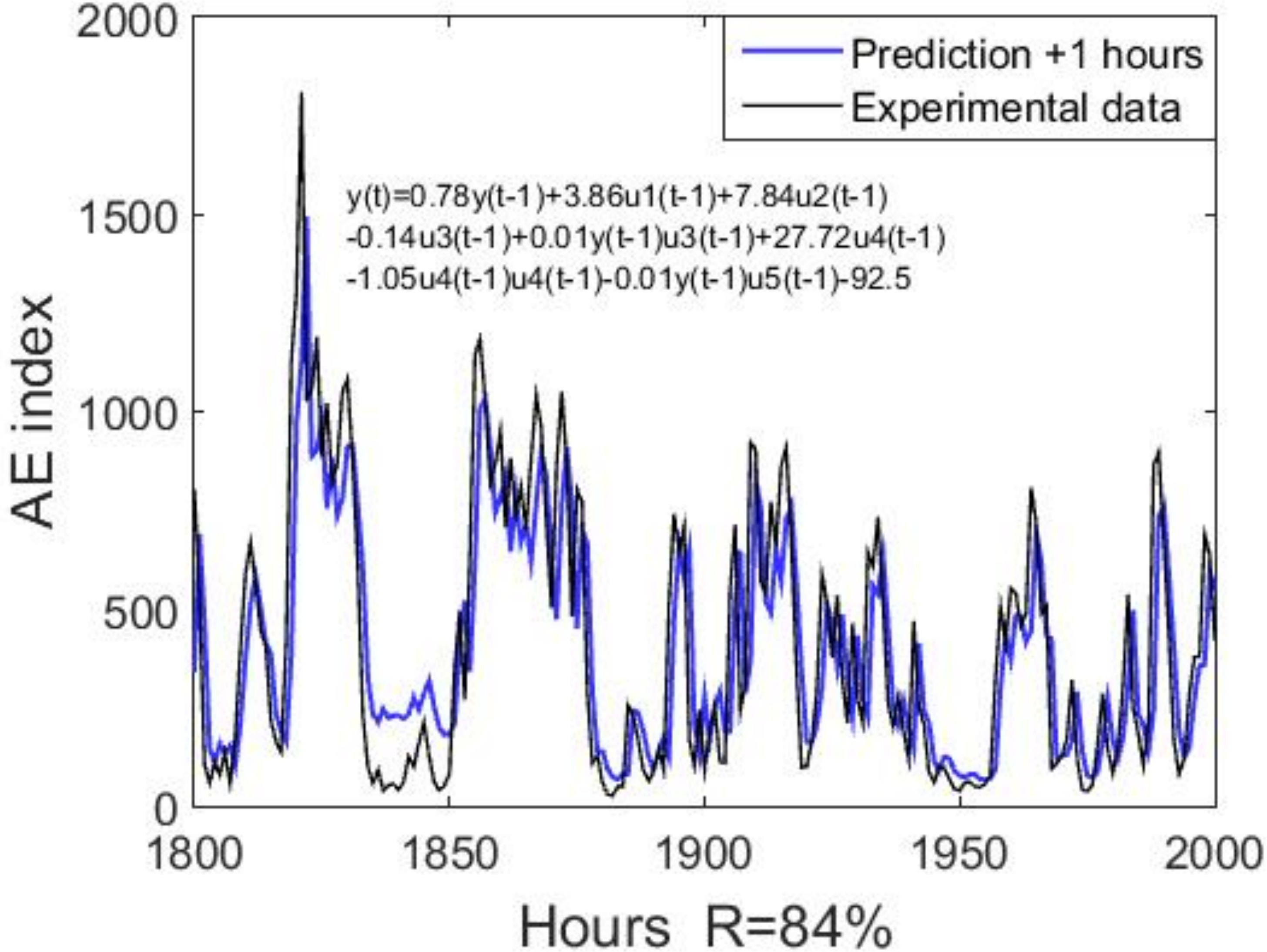
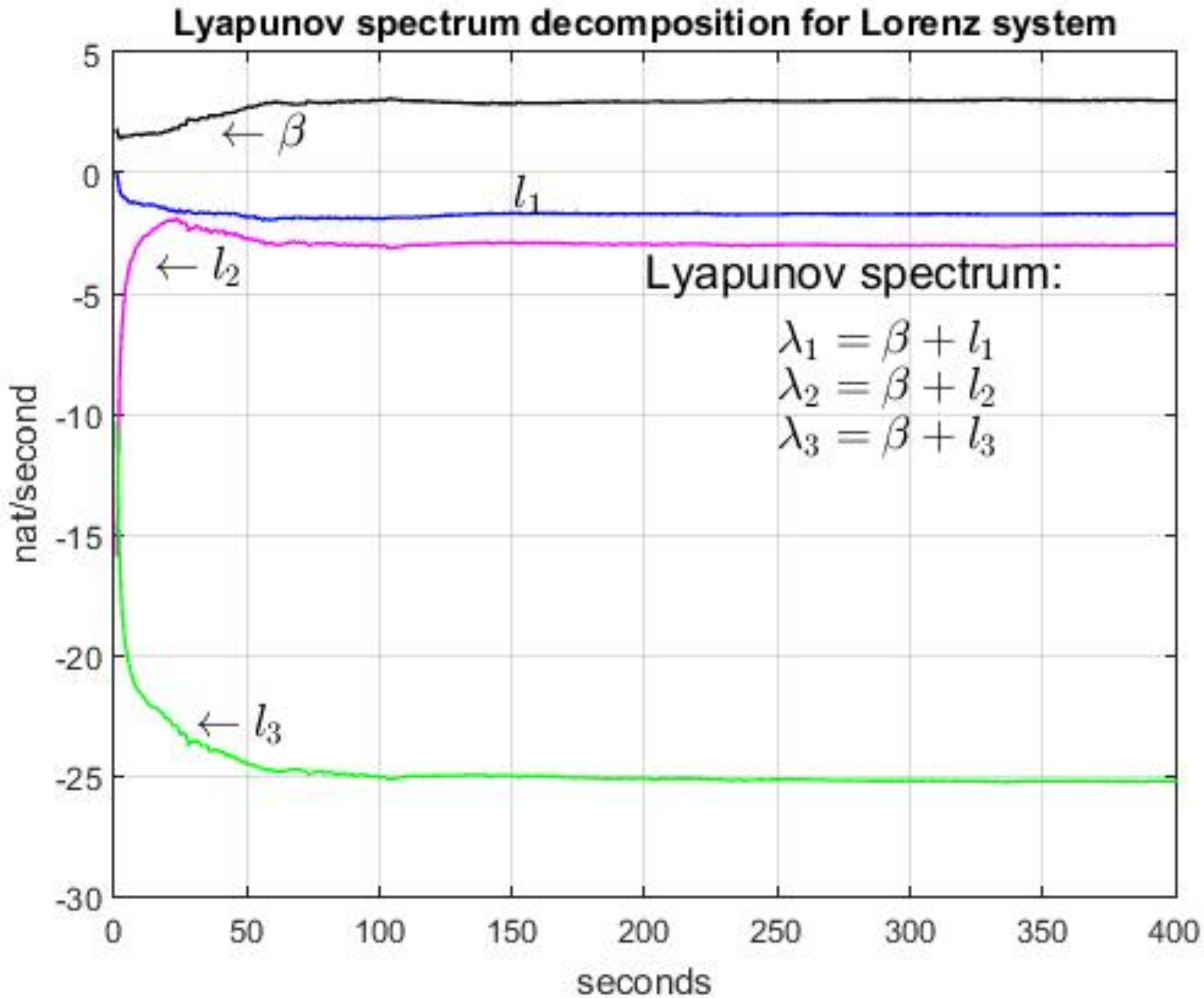


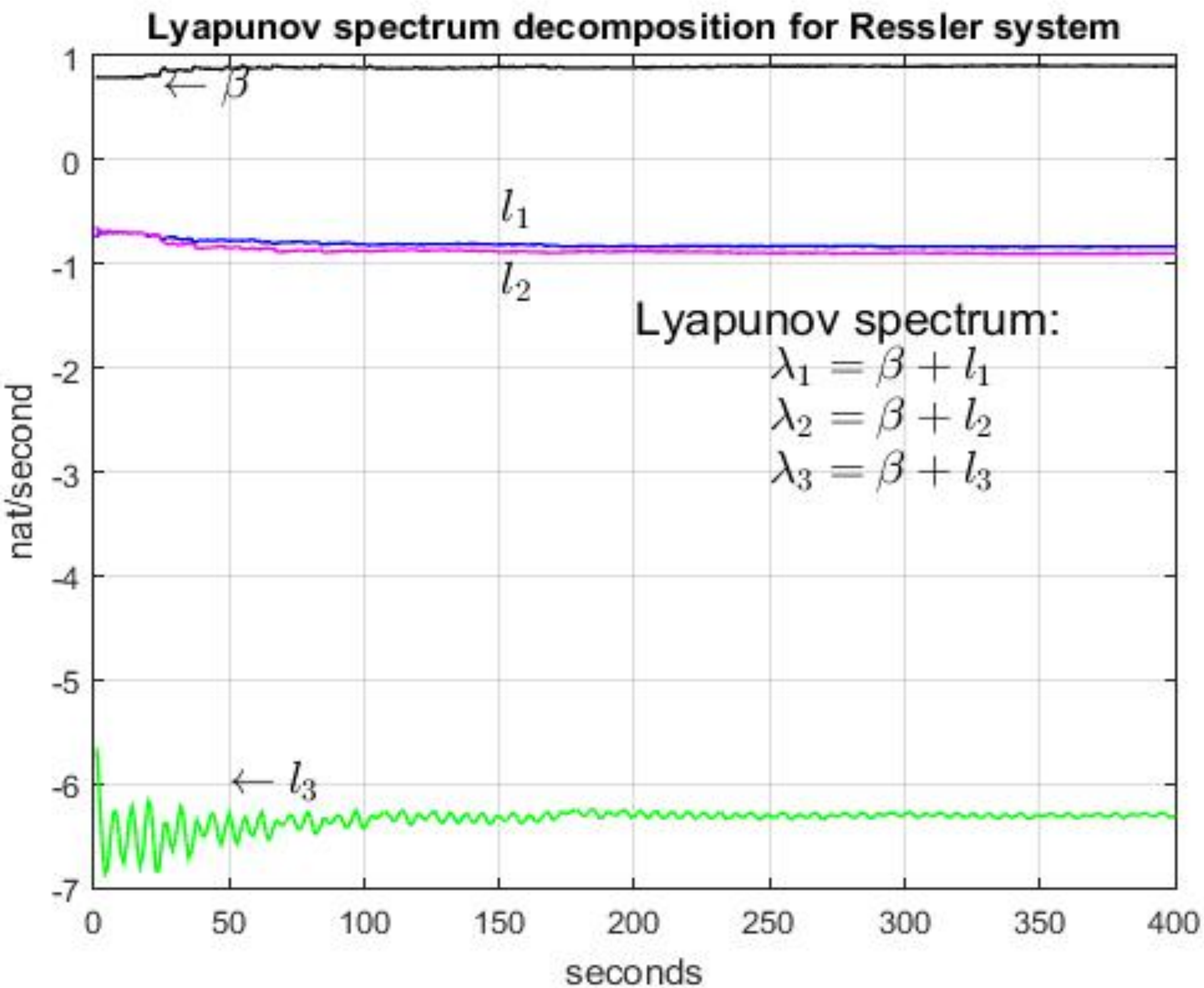
Fig. 3 AE index

Thanks
for your attention

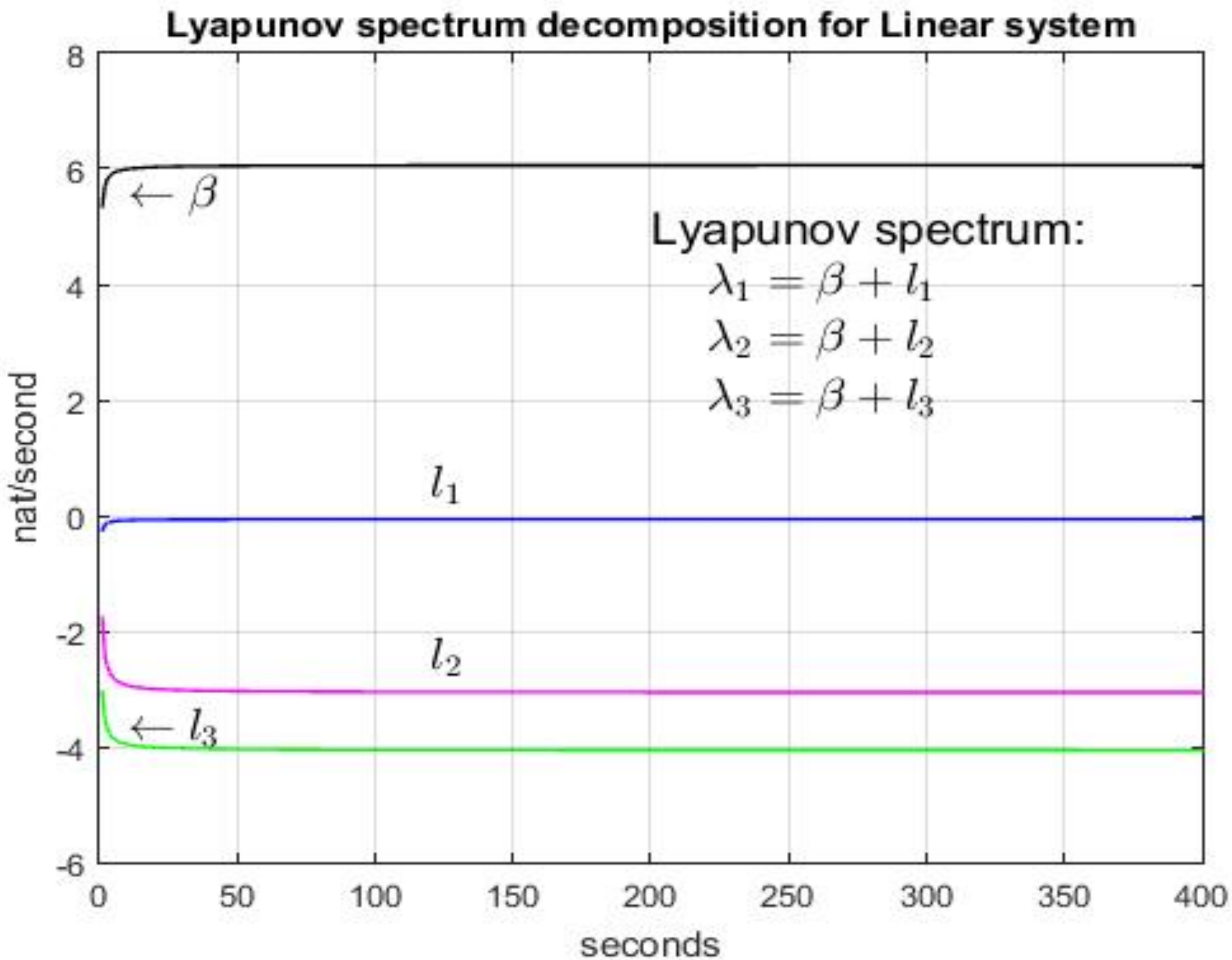
Чисельні результати для системи Лоренца



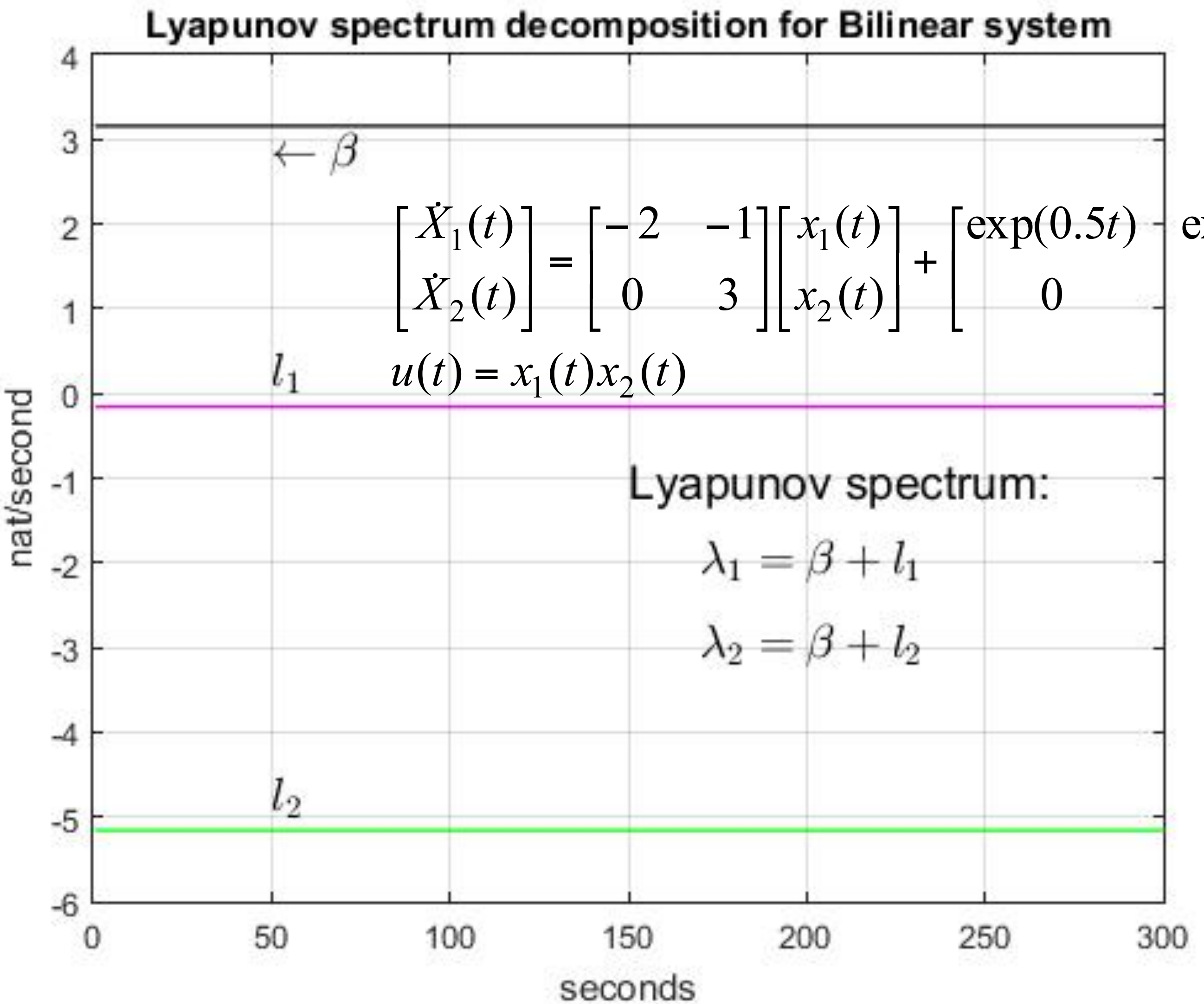
Чисельні результати для системи Ресслера



Чисельні результати для лінійної системи



Чисельні результати для білінійної системи



$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \exp(0.5t) & \exp(0.3t) \\ 0 & 0.5t \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} u(t),$$

$$u(t) = x_1(t) x_2(t)$$

Lyapunov spectrum:

$$\lambda_1 = \beta + l_1$$

$$\lambda_2 = \beta + l_2$$