



Abstract

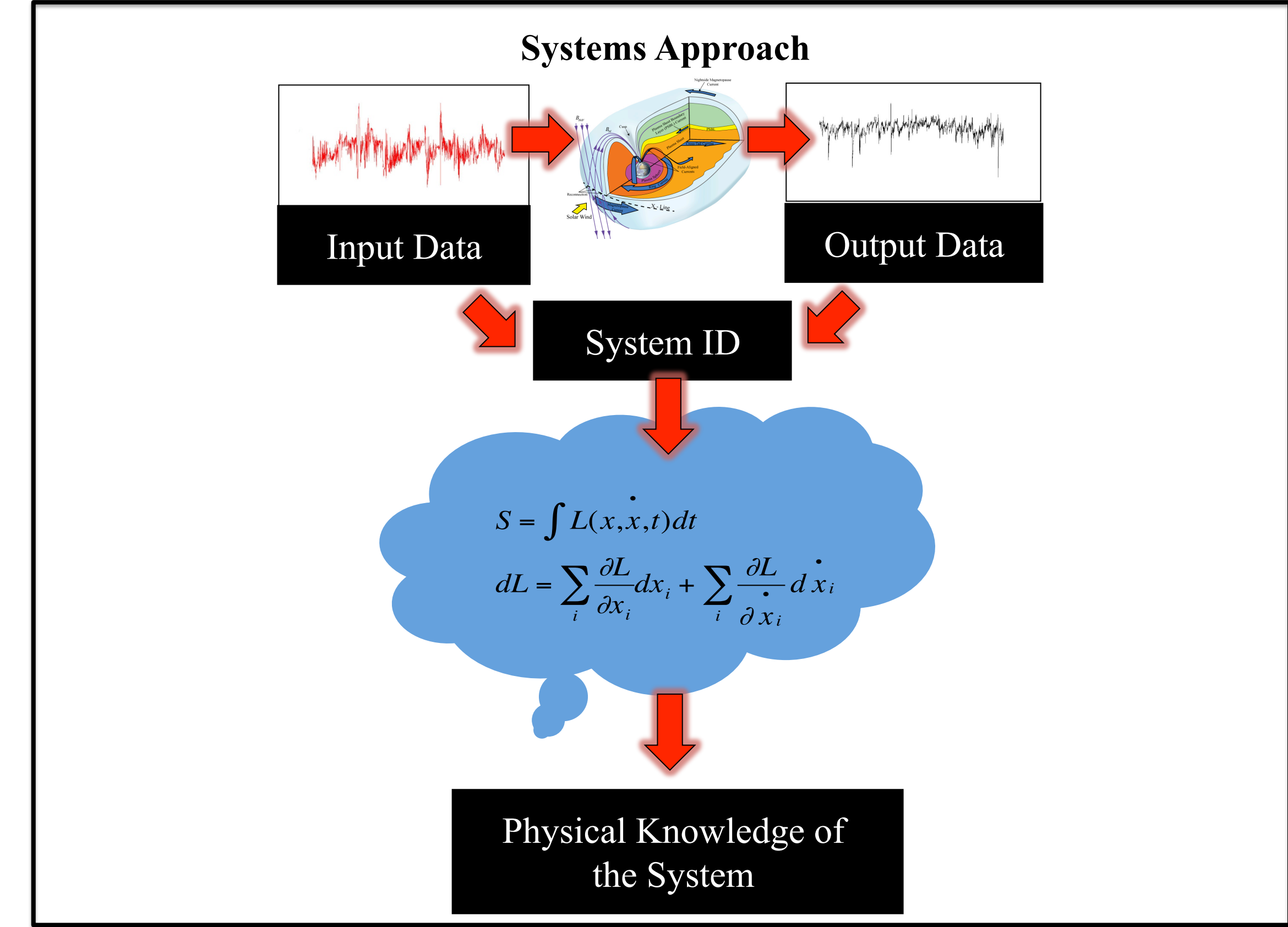
Forecast models have been developed and implemented online to provide forecasts of the energetic electrons at all energy ranges sampled by the third generation Geostationary Operational Environmental Satellites (GOES). These models are based on Multi-Input Single-Output (MISO) Nonlinear AutoRegressive Moving Average with eXogenous inputs (NARMAX) methodologies. The models use solar wind and geomagnetic indices as input data to produce a forecast of the energetic electrons at Geostationary Earth Orbit (GEO). These models have been running online since July 2015 and are shown to provide accurate forecasts that are capable of warning satellite operators of when the electrons at GEO could cause problems for their spacecraft. (<https://sbg.group.shef.ac.uk/progress2/html/index.phtml>).

Radiation Belts

The high fluence of these energetic electrons can cause a number of problems on spacecraft depending on the electron energy. For example, low energy electrons (10 keV to a few hundred keV) can cause surface charging that interferes with the satellite electronic systems. For higher energies (about 1 MeV and above) cause deep dielectric charging that may permanently damage the dielectric material onboard the satellite. Some of the effects of the energetic particles can be mitigated. However, this requires prior knowledge of high energetic electron populations that are dangerous to satellites. Models are required for these forecasts.

Modelling

System identification approach



NARMAX

Nonlinear **A**uto**R**egressive **M**oving **A**verage **e**Xogenous inputs

$$y(t) = F[y(t-1), \dots, y(t-n_y), u_1(t-1), \dots, u_1(t-n_{u_1}), \dots, u_m(t-1), \dots, u_m(t-n_{u_m}), e(t-1), \dots, e(t-n_e)] + e(t)$$

- Involves three stages
1. Structure selection:
 2. Coefficient estimation
 3. Model validation

Data

The NARMAX algorithm requires both input and output training data for the algorithm to deduce a model.

Model Training Data: The training data was from 1 March 2011 to 28 February 2013.

Inputs Data: Velocity, Density, pressure, the Dst Index, and $B_r \sin^6(\theta/2)$. The solar wind data were from the Advanced Composition Explorer (ACE) spacecraft positioned at the L1 Lagrange and supplied by the OMNI website for training the model. Dst was supplied by the World Data Center for Geomagnetism, Kyoto.

The past 24 hour averages were calculated hourly for each input. Therefore, the input time lags in the algorithm, n_{um} , were hourly. For example, the input $U(t-10)$ represents the average of the points between $U(t-10)$ hours and $U(t-34)$ hours.

The training data used lagged inputs from 2 to 48 hours.

Output Data: The output for each of the models are the daily averaged electron flux measurements taken from GOES MagED at GEO and are supplied by NOAA NWS Space Weather Prediction Center.

For the training data, the 1-minute corrected electron flux values were daily averaged between 00:01:00 UTC and 00:00:00 UTC the next day for each day. The training data employed autoregressive lags for the the previous 2 days, rather than hourly past 24 hour averages to avoid oversampling.

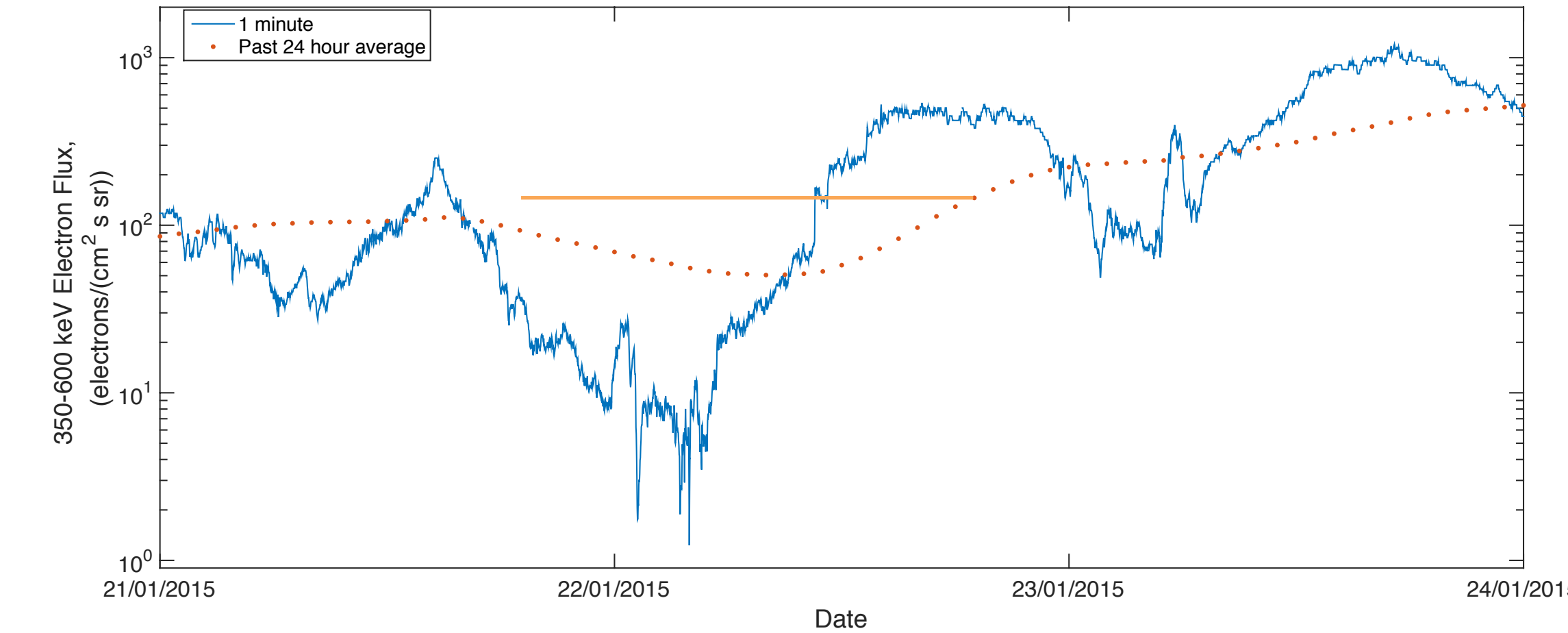
NARMAX model:

$$J(t) = F[J(t-24h), J(t-48h), v(t-2h), v(t-3h), \dots, v(t-48h), n(t-2h), n(t-3h), \dots, n(t-48h), p(t-2h), p(t-3h), \dots, p(t-48h), Dst(t-2h), Dst(t-3h), \dots, Dst(t-48h), B(t-2h), B(t-3h), \dots, B(t-48h), e(t-24h), e(t-48h)] + e(t)$$

Where F was a fourth degree polynomial.

Moving Average Data

After training the models on daily averaged output data, the 1-minute electron flux values were time averaged resulting in a data set with 1-hour resolution, such that each 1-hour point was determined by averaging the 5-minute data over the past 24 hours, e.g., the point at 19:00:00 UTC on 22 January 2015 is average of the 288 5-minute points between 19:01:00 UTC on 21 January 2015 and 19:00:00 UTC on 22 January 2015. This data would then be compared to the model forecast.



Statistical Analysis of the Models Performance

Prediction Efficiency

$$PE = 1 - \frac{\sum_{t=1}^N [(y(t) - \hat{y}(t))^2]}{\sum_{t=1}^N [(y(t) - \bar{y}(t))^2]}$$

Where $y(t)$ is the measured output at time t , \hat{y} is the forecast output, N is the length of the data and the bar indicates the mean. The PE and CC were calculated for each of the model forecasts over the time period shown in the Table below

Correlation coefficient

$$CC = \frac{\sum_{t=1}^N [(y(t) - \bar{y}(t))(\hat{y}(t) - \bar{\hat{y}}(t))]}{\sqrt{\sum_{t=1}^N [(y(t) - \bar{y}(t))^2] \sum_{t=1}^N [(\hat{y}(t) - \bar{\hat{y}}(t))^2]}}$$

Forecast Time

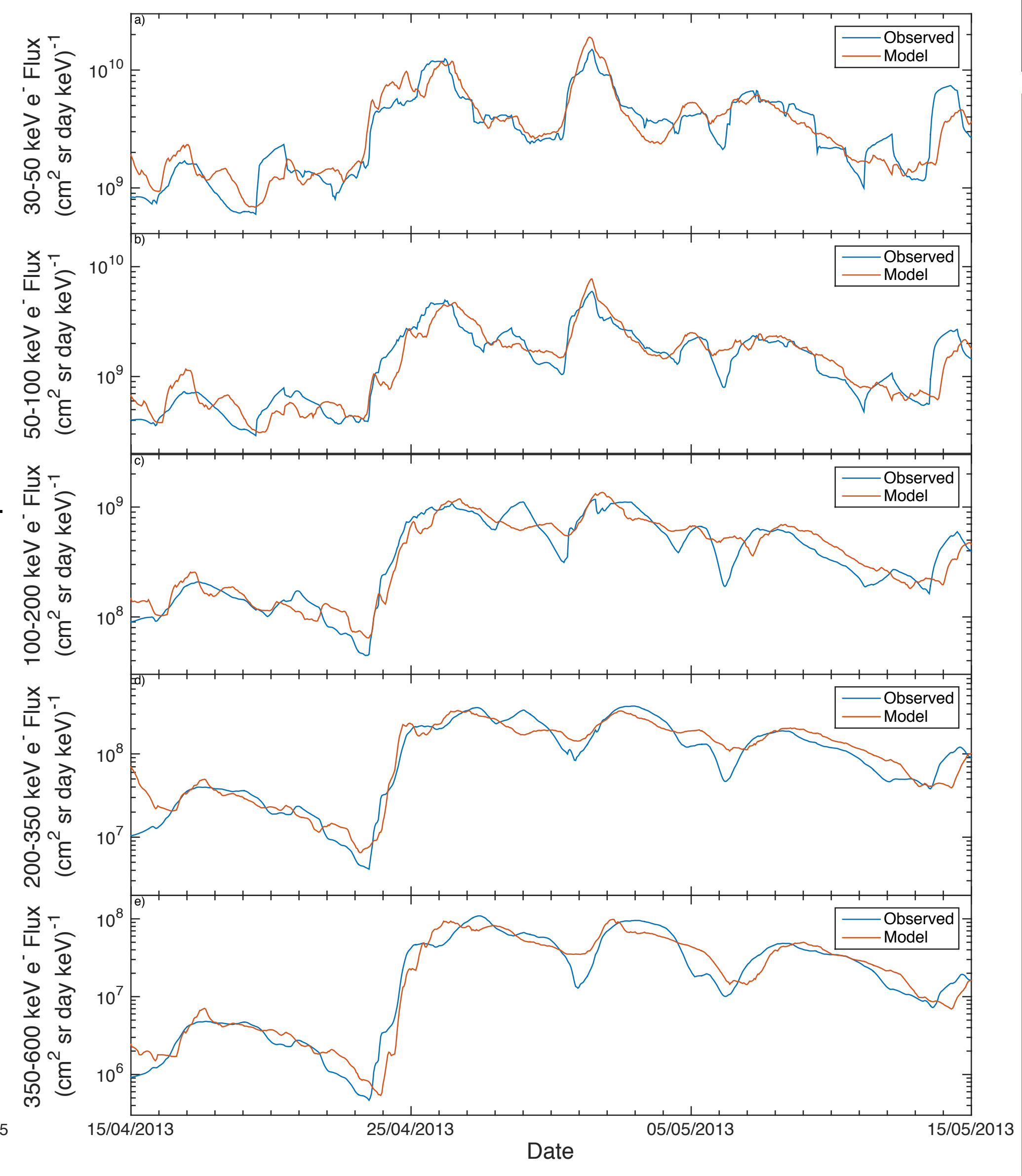
The amount of time that the NARMAX model is able to forecast into the future is dependent on the minimum lag within the final NARMAX model.

For example, if the minimum lag within the NAMAX model is a velocity value 10 hours ago

$$J(t) = aV(t-10) + \dots$$

Where a is the coefficient, then if we know the velocity at the present time t , then we can calculate an estimate of the electron flux, J , at time $t+10$ hours (a 10 hour ahead forecast)

Model Performance Figures



Model	Forecast Time (hours)	PE (%)	CC (%)
40-50 keV	10	66.9	82.0
50-100 keV	12	69.2	83.5
100-200 keV	16	73.2	85.6
200-350 keV	24	71.6	84.9
350-300 keV	24	73.6	85.9

Conclusions

The aim of this study was to create forecast models for the electron flux energy ranges observed by the third generation GOES satellites.

NARMAX is the most robust method for probing nonlinear processes in data.

All of these models will be implemented in real time to forecast the electron fluxes on the University of Sheffield Space Weather website: www.sbg.group.shef.ac.uk/USSW/UOSSW.html

